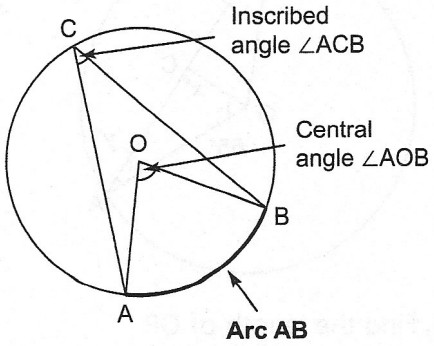
8.3 Properties of Angles in a Circle

FOCUS **Use inscribed angles and central angles to solve problems.**

In a circle:

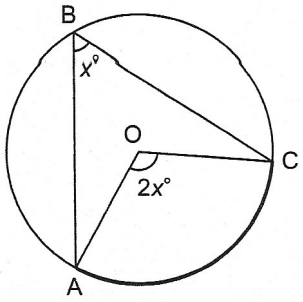
- A **central angle** has its vertex at the centre.
- An **inscribed angle** has its vertex on the circle.

Both angles in the diagram are **subtended** by **arc** AB.



Central Angle and Inscribed Angle Property

The measure of a central angle is twice the measure of an inscribed angle subtended by the same arc.



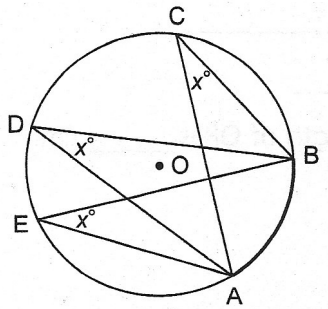
So, $\angle AOC = 2\angle ABC$, or

$$\angle ABC = \frac{1}{2}\angle AOC$$

Inscribed Angles Property

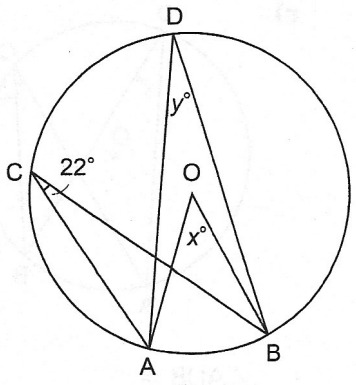
Inscribed angles subtended by the same arc are equal.

So, $\angle ACB = \angle ADB = \angle AEB$

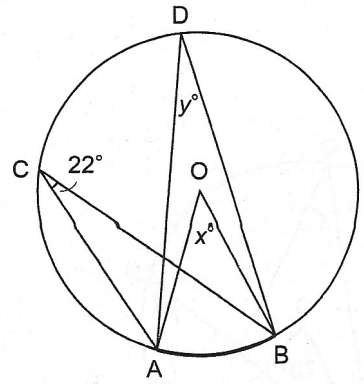


Example 1 Using Inscribed and Central Angles

Find the values of $x°$ and $y°$.



Solution



Central $\angle AOB$ and inscribed $\angle ACB$ are both subtended by arc AB.

So, $\angle AOB = 2\angle ACB$

$x° = 2 \times 22°$

$= 44°$

$\angle ACB$ and $\angle ADB$ are inscribed angles subtended by the same arc AB.

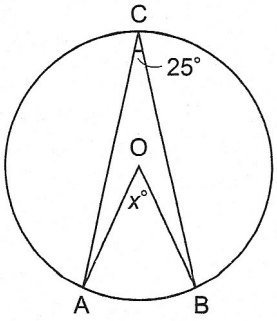
So, $\angle ADB = \angle ACB$

$y° = 22°$

Check

1. Find each value of $x°$.

a)

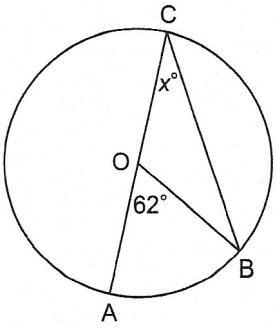


$\angle AOB = 2 \times \angle ACB$

$x° = 2 \times$ _____

$=$ _____

b)

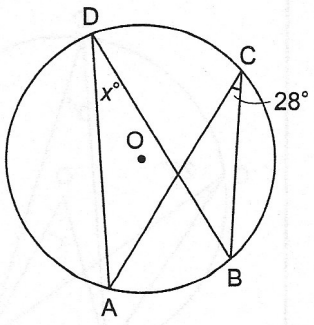


$\angle ACB = \frac{1}{2} \times$ _____

$x° = \frac{1}{2} \times$ _____

$=$ _____

c)

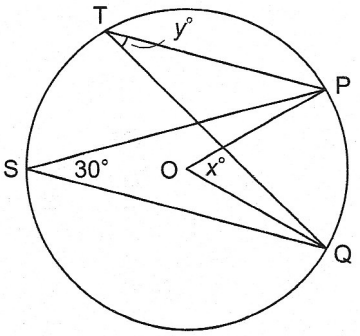


$\angle ADB =$ _____

$x° =$ _____

2. Find the values of $x°$ and $y°$.

a)



$\angle QOP = 2 \times \angle QSP$

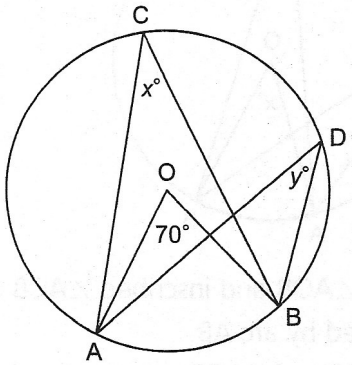
$x° = 2 \times$ _____

$x° =$ _____

$\angle QTP =$ _____

$y° =$ _____

b)



$\angle ACB = \frac{1}{2} \times$ _____

$x° = \frac{1}{2} \times$ _____

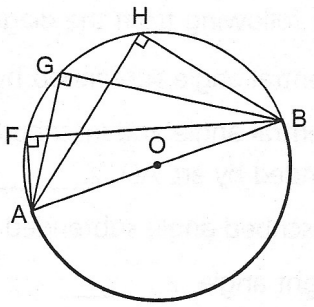
$x° =$ _____

$\angle ADB =$ _____

$y° =$ _____

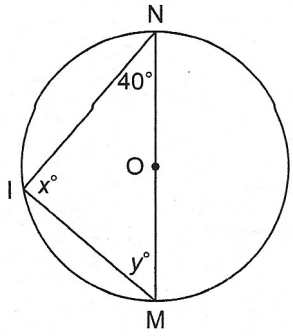
Angles in a Semicircle Property

Inscribed angles subtended by a semicircle are right angles.
$\angle AFB = \angle AGB = \angle AHB = 90°$



Example 2 Finding Angles in an Inscribed Triangle

Find $x°$ and $y°$.



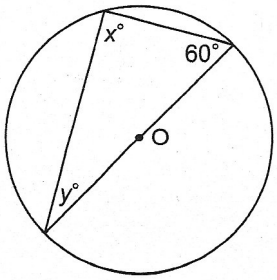
Solution

$\angle MIN$ is an inscribed angle subtended by a semicircle.
So, $x° = 90°$

$y° = 180° - 90° - 40°$ By the angle sum property in $\triangle MIN$
$= 50°$

Check

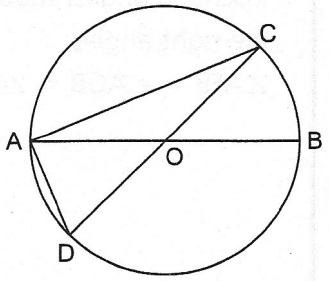
1. Find the values of $x°$ and $y°$.



$x° =$ _____
$y° = 180° -$ _____ $-$ _____
$y° =$ _____

Practice

1. Name the following from the diagram.



a) the central angle subtended by arc CB: ∠_____

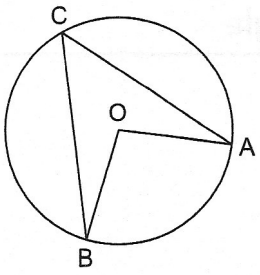
b) the central angle and inscribed angle subtended by arc AD: ∠_____ and ∠_____

c) the inscribed angle subtended by a semicircle: ∠_____

d) the right angle: ∠_____

2. In each circle, name a central angle and an inscribed angle subtended by the same arc. Shade the arc.

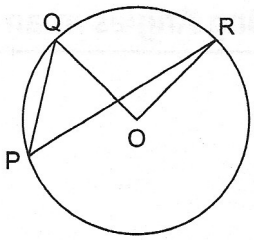
a)



Central angle: ∠_____

Inscribed angle: ∠_____

b)

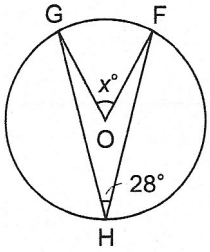


Central angle: ∠_____

Inscribed angle: ∠_____

3. Determine each indicated measure.

a)

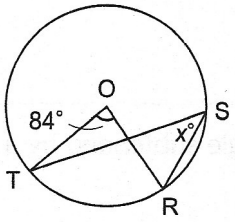


∠GOF = 2 × ∠GHF

$x° = 2 \times$ _____

= _____

b)

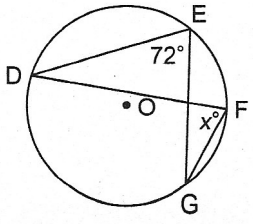


$\angle TSR = \frac{1}{2} \times \angle$_____

$x° = \frac{1}{2} \times$ _____

$x°$ = _____

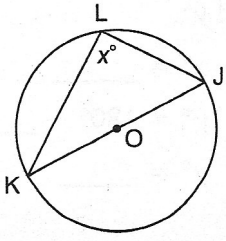
c)



∠DEG = _____

$x°$ = _____

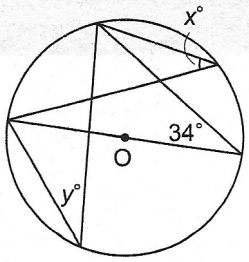
d)



$x°$ = _____

4. Determine each value of $x°$ and $y°$.

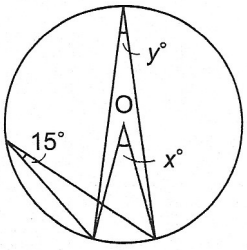
a)



$x° =$ _____

$y° =$ _____

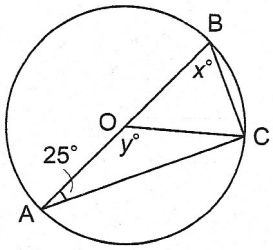
b)



$x° =$ _____ × _____ = _____

$y° =$ _____

5. Find the value of $x°$ and $y°$.



$\angle ACB =$ _____

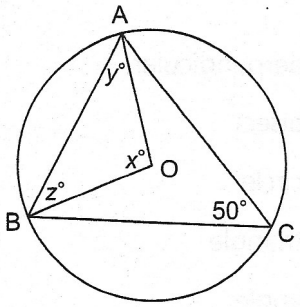
$x° = 180° -$ _____ − _____ By the angle sum property

$=$ _____

$y° =$ _____ × _____

$=$ _____

6. Find the value of $x°$, $y°$, and $z°$.



$\angle AOB = 2 \times$ _____

$x° = 2 \times$ _____ = _____

In $\triangle OAB$, _____ = _____

$\triangle OAB$ is _____.

In $\triangle OAB$:

$y° = z°$

$y° + y° =$ _____ − _____ By the angle sum property

$2y° =$ _____

$y° = \frac{\quad}{2}$

So, $y° =$ _____ and $z° =$ _____