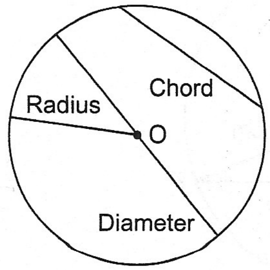
8.2 Properties of Chords in a Circle

FOCUS Use chords and related radii to solve problems.

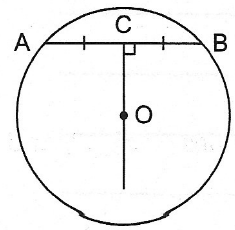
A **chord** of a circle joins 2 points on the circle.



Chord Properties

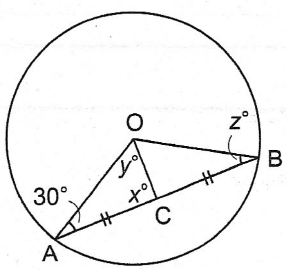
In any circle with centre O and chord AB:

- If OC bisects AB, then OC ⊥ AB.
- If OC ⊥ AB, then AC = CB.
- The perpendicular bisector of AB goes through the centre O.



Example 1 Finding the Measure of Angles in a Triangle

Find $x°$, $y°$, and $z°$.



Solution

OC bisects chord AB, so OC ⊥ AB

Therefore, $x° = 90°$

By the angle sum property in △OAC:

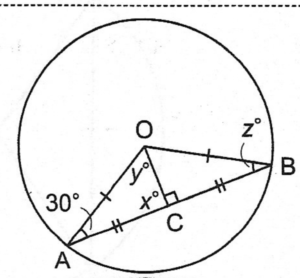
$y° = 180° - 90° - 30°$

$= 60°$

Since radii are equal, OA = OB, and △OAB is isosceles.

∠OBA = ∠OAB

So, $z° = 30°$



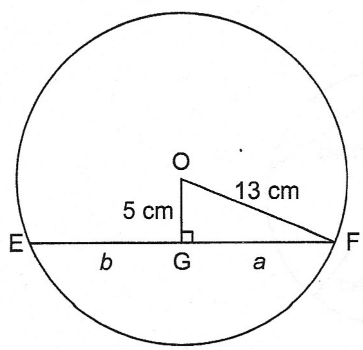
In an isosceles triangle, 2 base angles are equal.

Check

1. Find the values of *a* and *b*.

5

2



2

____ = ____ + a^2 By the Pythagorean Theorem in right △OFG

____

____

____

____

____

So, a = ____ cm

____ = ____ By the chord properties

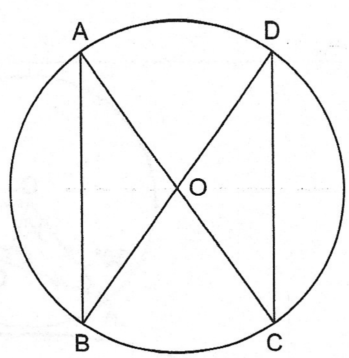
So, b = ____ cm

Practice

In each diagram, O is the centre of the circle.

1. Name all radii, chords, and diameters.

a)

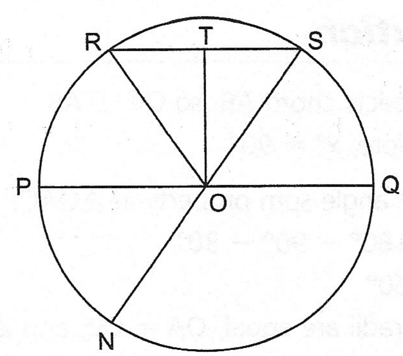


Radii: ______________

Chords: ______________

Diameters: ______________

b)



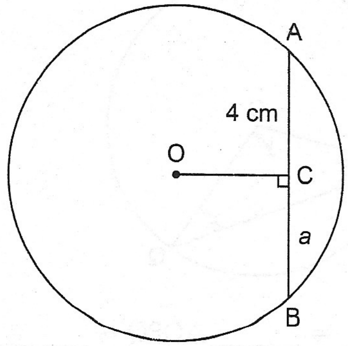
Radii: ______________

Chords: ______________

Diameters: ______________

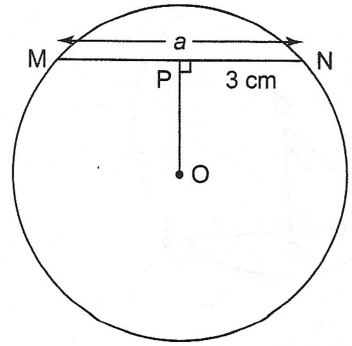
2. On each diagram, mark line segments with equal lengths.
Then find each value of a.

a)



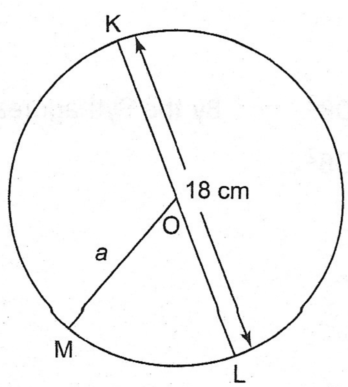
AC = CB = ______ cm
So, a = ______ cm

b)



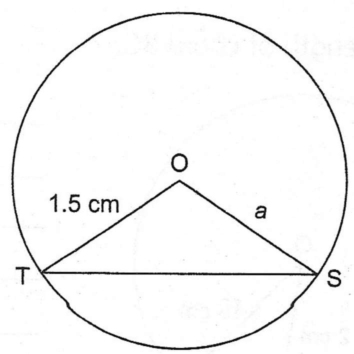
MN = 2 × ______
= 2 × ______ cm
= ______ cm
So, a = ______

c)



OL = $\frac{1}{2}$ × ______
= $\frac{1}{2}$ × ______ cm
= ______ cm
So, a = ______

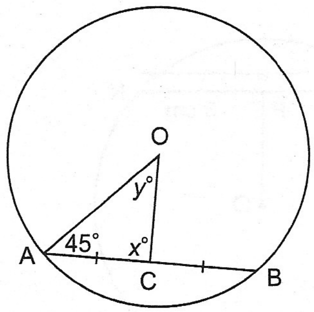
d)



OS = ______ = ______ cm
So, a = ______

3. Find each value of $x°$ and $y°$.

a)

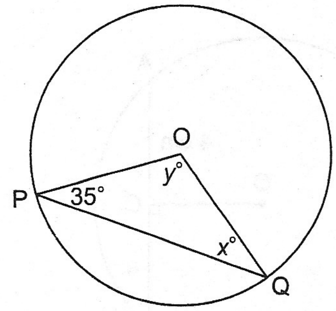


$x° =$ _____

$y° = 180° -$ _____ $-$ _____

$=$ _____

b)



_____ $=$ _____ $\triangle$OPQ is _____________

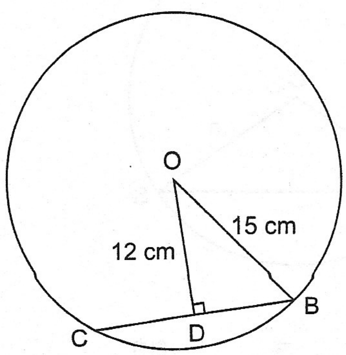
$\angle$_____ $= \angle$_____

So, $x° =$ _____

$y° = 180° -$ _____ $-$ _____

$=$ _____

4. Find the length of chord BC.



_____ $=$ _____ $+ \text{DB}^2$ By the Pythagorean Theorem.

_____ $=$ _____ $+ \text{DB}^2$

____

____

____

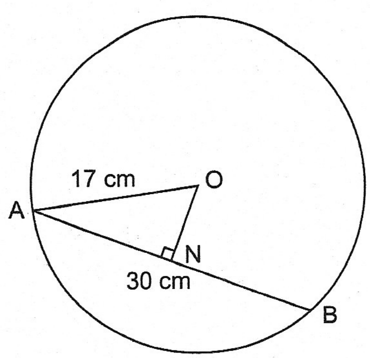
____

So, DB $=$ _____ cm

_____ $=$ _____ $=$ _____ cm By the chord properties

So, chord BC has length: $2 \times$ _____ cm $=$ _____ cm

5. Find ON.



AN $= \frac{1}{2} \times$ _____

$= \frac{1}{2} \times$ _____ cm By the chord properties

$=$ _____ cm

_____ $=$ _____ $+ \text{ON}^2$ By the Pythagorean Theorem

_____ $=$ _____ $+ \text{ON}^2$

____

____

____

____

____

So, ON is _____ cm.

CHECKPOINT

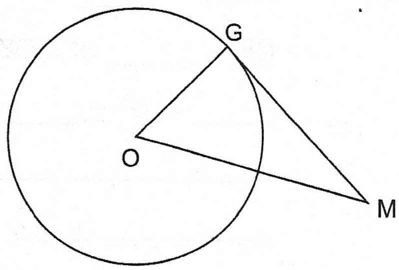
Can you ...

- Solve problems using tangent properties?
- Solve problems using chord properties?

8.1 In each diagram, O is the centre of the circle.
Assume that lines that appear to be tangent are tangent.

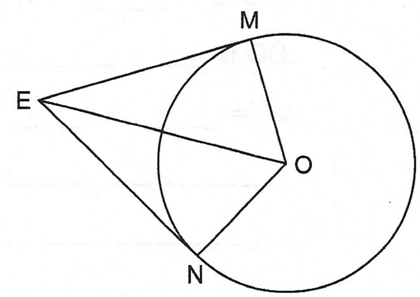
1. Name the angles that measure 90°.

a)



____

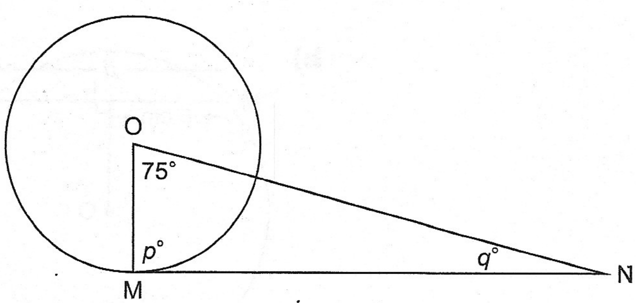
b)



____

2. Find the unknown angle measures.

a)

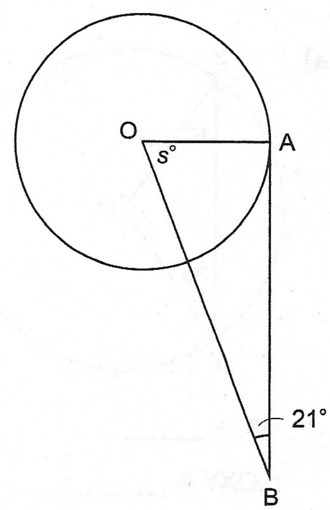


$p° =$ ____ Tangent-radius property

$q° = 180° -$ ____ $-$ ____ Angle sum property

$q° =$ ____

b)



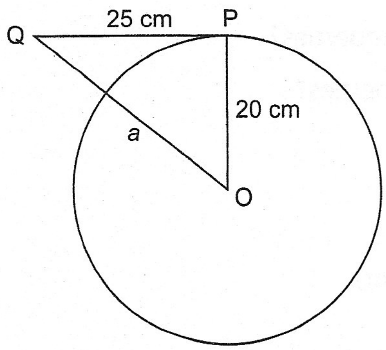
____ $= 90°$

$s° =$ ____ $-$ ____ $-$ ____

$s° =$ ____

3. Find the values of *a* and *b* to the nearest tenth.

a)



$\angle$OPQ = ______ By the tangent-radius property

OQ is ________________ of $\triangle$OPQ.

a^2 = ______ + ______ By the Pythagorean Theorem

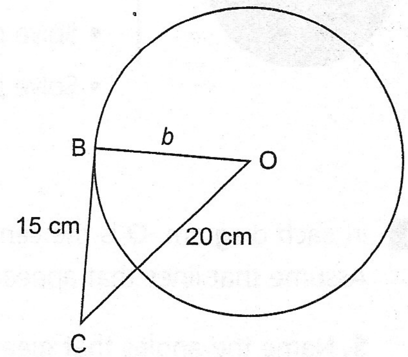
____

____

____

So, $a \doteq$ ______ cm

b)



$\angle$OBC = ______

OB is ______ of $\triangle$OBC.

______ = b^2 + ______

____

____

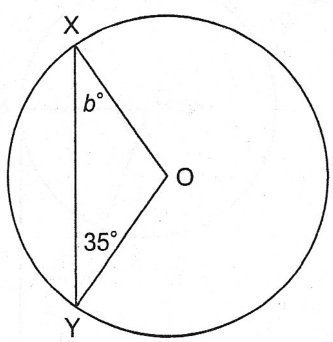
____

____

So, $b \doteq$ ______ cm

8.2 **4.** Find the unknown measures.

a)

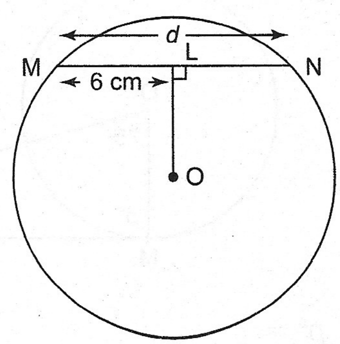


OX = ______

$\triangle$OXY is __________.

So, $b°$ = ______

b)



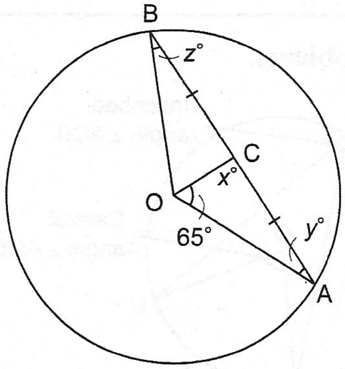
MN = 2 × ______

MN = ______ × ______ cm

= ______ cm

So, *d* = ______ cm

5. Find each value of $x°$, $y°$, and $z°$.



$x° =$ ______ By the chord properties

$y° =$ ______ $-$ ______ $-$ ______ By the angle sum property

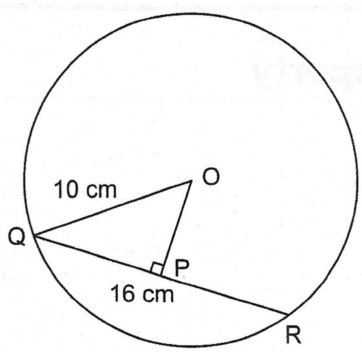
$=$ ______

______ $=$ ______, so ______ is isosceles.

$\angle$______ $= \angle$______

So, $z° =$ ______

6. Find the length of OP.



$QP = \frac{1}{2} \times QR$ By the chord properties

$= \frac{1}{2} \times$ ______ cm

$=$ ______ cm

$OQ^2 =$ ______ $+ OP^2$ By the Pythagorean Theorem

______ $=$ ______ $+ OP^2$

____

____

____

____

$=$ ______

So, the length of OP is ______ cm.