UNIT 8

Circle Geometry

What You'll Learn

How to

- Solve problems involving tangents to a circle
- Solve problems involving chords of a circle
- Solve problems involving the measures of angles in a circle

Why Is It Important?

Circle properties are used by .

- artists, when they create designs and logos

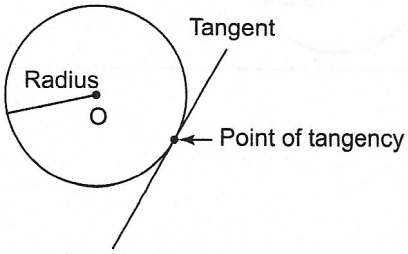
Key Words

radius (radii)
right angle
tangent
point of tangency
diameter
right triangle
isosceles triangle
chord
perpendicular bisector
central angle
inscribed angle
arc
subtended
semicircle

8.1 Properties of Tangents to a Circle

FOCUS **Use the relationship between tangents and radii to solve problems.**

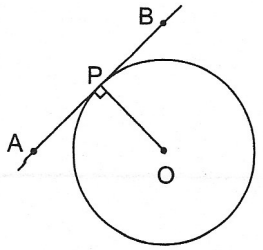
A **tangent** touches a circle at exactly one point.



Tangent-Radius Property

A tangent to a circle is perpendicular to the radius drawn to the point of tangency.

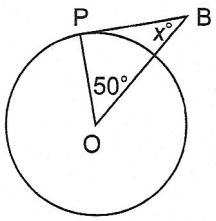
So, OP ⊥ AB, ∠OPA = 90° and ∠OPB = 90°



⊥ means "perpendicular to".

Example 1 Finding the Measure of an Angle in a Triangle

BP is tangent to the circle at P.
O is the centre of the circle.
Find the measure of $x°$.



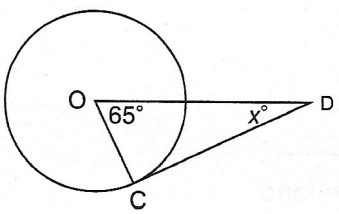
Solution

By the tangent-radius property: ∠OPB = 90°
Since the sum of the angles in △OPB is 180°:
$x° = 180° - 90° - 50°$
$= 40°$
So, $x°$ is 40°.

Check

1. Find the value of $x°$.

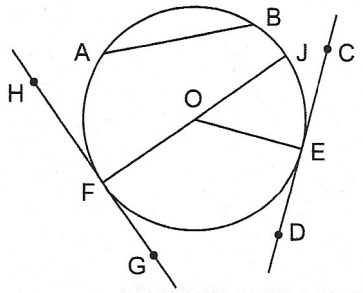
∠_____ = 90°
$x° = 180° -$ _____ − _____
= _____



Practice

In each question, O is the centre of the circle.

1. From the diagram, identify:



a) 3 radii _____, _____, _____

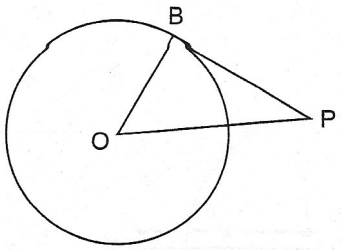
b) 2 tangents _____, _____

c) 2 points of tangency _____, _____

d) 4 right angles ∠_____, ∠_____, ∠_____, ∠_____

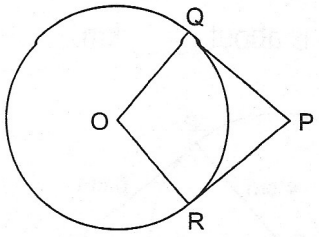
2. What is the measure of each angle?

a)



∠OBP = _____

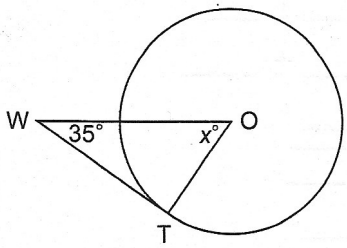
b)



∠PQO = _____ ∠PRO = _____

3. Find each value of $x°$.

a)

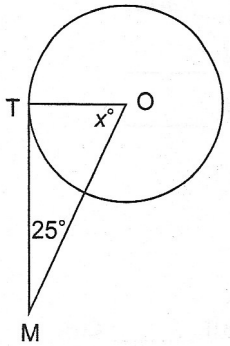


∠OTW = _____

$x°$ = 180° − _____ − _____

= _____

b)



_____

_____

= _____

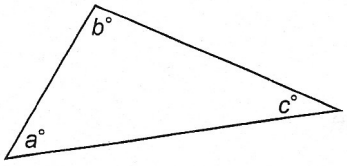
8.1 Skill Builder

Solving for Unknown Measures in Triangles

Here are 2 ways to find unknown measures in triangles.

Angle Sum Property

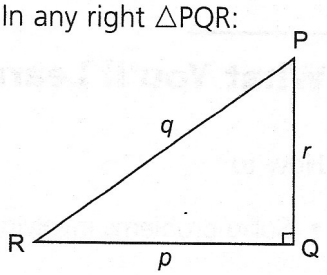
In any triangle:



$a° + b° + c° = 180°$

Pythagorean Theorem

In any right △PQR:

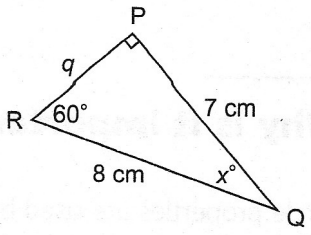


$q^2 = p^2 + r^2$

Here is how to find the unknown measures in right △PQR.

In △PQR, the angles add up to 180°.
To find $x°$, start at 180° and subtract the known measures.

$x° = 180° - 90° - 60°$
$= 30°$



By the Pythagorean Theorem:

$QR^2 = PR^2 + PQ^2$
$8^2 = q^2 + 7^2$
So: $q^2 = 8^2 - 7^2$
$q = \sqrt{8^2 - 7^2}$
$\doteq 3.87$

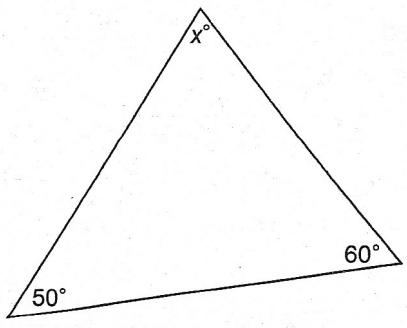
So, $x°$ is 30° and q is about 4 cm.

Answer to the same degree of accuracy as the question uses.

Check

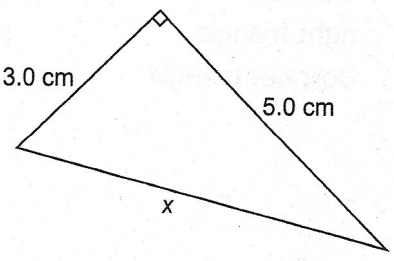
1. Find each unknown measure.

a)



$x° = 180° - \_\_\_\_\_ - \_\_\_\_\_$
$= \_\_\_\_\_$

b)

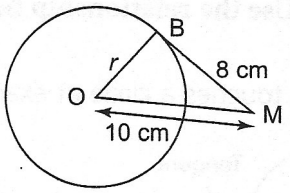


$x^2 = \_\_\_\_\_ + \_\_\_\_\_$
$x = \sqrt{\_\_\_\_\_ + \_\_\_\_\_}$
$\doteq \_\_\_\_\_$

So, x is _____.

Example 2 Using the Pythagorean Theorem in a Circle

MB is a tangent to the circle at B. O is the centre.
Find the length of radius OB.



Solution

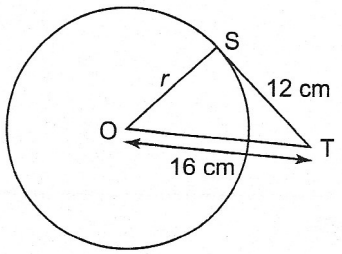
By the tangent-radius property: $\angle OBM = 90°$
By the Pythagorean Theorem in right $\triangle MOB$:

$$OM^2 = OB^2 + BM^2$$
$$10^2 = r^2 + 8^2$$
$$100 = r^2 + 64$$
$$100 - 64 = r^2$$
$$36 = r^2$$
$$\sqrt{36} = r$$
$$r = 6$$

Radius OB has length 6 cm.

Check

1. ST is a tangent to the circle at S. O is the centre.
Find the length of radius OS.
Answer to the nearest millimetre.



$\angle OST =$ _____ By the tangent-radius property

$OT^2 =$ _____ + _____ By the Pythagorean Theorem

_____ = r^2 + _____

_____ = r^2 + _____

_____ − _____ = r^2

_____ = r^2

$\sqrt{\text{_____}} = r$

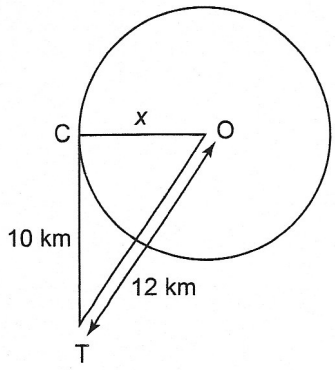
$r \doteq$ _____

OS is about _____ cm long.

4. Find each value of x.

Answer to the nearest tenth of a unit.

a)



$\angle OCT = 90°$ By the tangent-radius property

$\_\_\_\_ = x^2 + \_\_\_\_$ By the Pythagorean Theorem in $\triangle OCT$

$\_\_\_\_ = x^2 + \_\_\_\_$

$\_\_\_\_ - \_\_\_\_ = x^2$

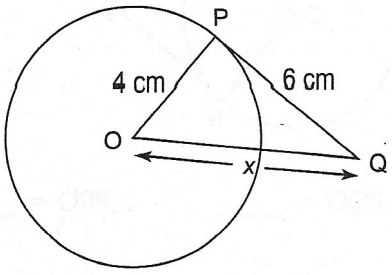
$\_\_\_\_ = x^2$

$\sqrt{\_\_\_\_} = x$

$x \doteq \_\_\_\_$

So, OC is about _____ km.

b)



$\angle OPQ =$ _____, and:

$x^2 = \_\_\_\_ + \_\_\_\_$

$x^2 = \_\_\_\_ + \_\_\_\_$

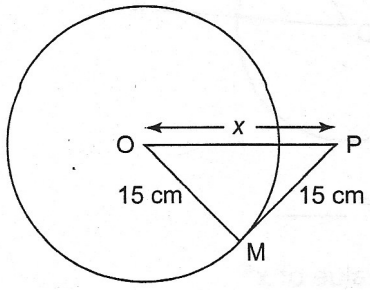
$x^2 = \_\_\_\_$

$x = \sqrt{\_\_\_\_}$

$x \doteq \_\_\_\_$

So, OQ is about _____ cm.

c)



$x^2 = \_\_\_\_ + \_\_\_\_$

$x^2 = \_\_\_\_ + \_\_\_\_$

$x^2 = \_\_\_\_$

$x = \sqrt{\_\_\_\_}$

$x \doteq \_\_\_\_$

So, OP is about _____ cm.