7.4 Skill Builder

Sum of the Angles in a Triangle

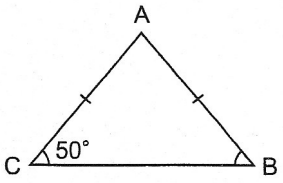
In any triangle, the sum of the angle measures is 180°.

So, to find an unknown angle measure:

- start with 180°
- subtract the known measures

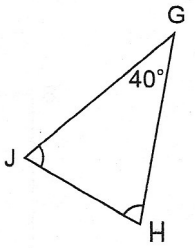
An isosceles triangle has 2 equal sides and 2 equal angles.

To find the measure of the third angle, subtract the measure of the equal angles twice.



$\angle A = 180° - 50° - 50°$
$= 80°$

To find the measure of each equal angle, subtract the known angle from 180°, then divide by 2.

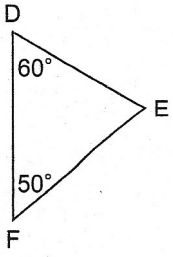


Sum of equal angles is: $180° - 40° = 140°$
Measure of each equal angle: $140° \div 2 = 70°$

Check

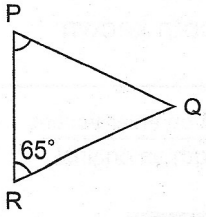
1. Find the measure of the third angle.

a)



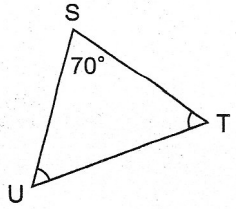
$\angle E = 180° - \_\_\_\_ - \_\_\_\_$
$= \_\_\_\_$

b)



$\angle Q = \_\_\_\_\_$
$= \_\_\_\_$

2. Find the measure of each equal angle.



Sum of equal angles is:
$180° - \_\_\_\_ = \_\_\_\_$
Measure of each equal angle:
$\_\_\_\_ \div 2 = \_\_\_\_$

7.4 Similar Triangles

FOCUS **Use the properties of similar triangles to solve problems.**

A triangle is a special polygon.
When two triangles are similar:

- matching angles are equal **OR**
- matching sides are proportional

The order in which similar triangles are named gives a lot of information.

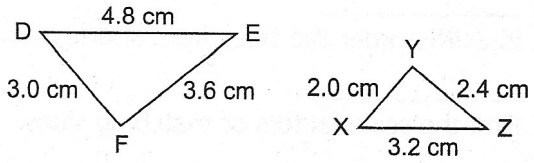
Suppose $\triangle ABC \sim \triangle DEF$.

The symbol ~ means "is similar to."

Then, $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$
Similarly, AB matches DE, BC matches EF, and AC matches DF.

Example 1 Identifying Similar Triangles

Name the similar triangles.



Solution

Angle measures are not given.
So, find out if matching sides are proportional.
In $\triangle DEF$, order the sides from shortest to longest: FD, EF, DE
In $\triangle XYZ$, order the sides from shortest to longest: XY, YZ, ZX
Find the scale factors of matching sides.

$$\frac{\text{length of FD}}{\text{length of XY}} = \frac{3.0 \text{ cm}}{2.0 \text{ cm}} = 1.5$$

$$\frac{\text{length of EF}}{\text{length of YZ}} = \frac{3.6 \text{ cm}}{2.4 \text{ cm}} = 1.5$$

$$\frac{\text{length of DE}}{\text{length of ZX}} = \frac{4.8 \text{ cm}}{3.2 \text{ cm}} = 1.5$$

Since all scale factors are the same, the triangles are similar.

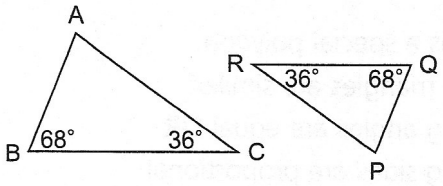
The longest and shortest sides meet at vertices: D and X
The two longer sides meet at vertices: E and Z
The two shorter sides meet at vertices: F and Y
So, $\triangle DEF \sim \triangle XZY$

Read the letters down the columns.

Check

1. In each diagram, name two similar triangles.

a) Two angles in each triangle are given.
The measure of the third angle
in each triangle is:
180° − _____



List matching angles:
∠A = _____ = _____
∠B = _____ = _____
∠C = _____ = _____
Matching angles _____ equal.
So, the triangles _____ similar.

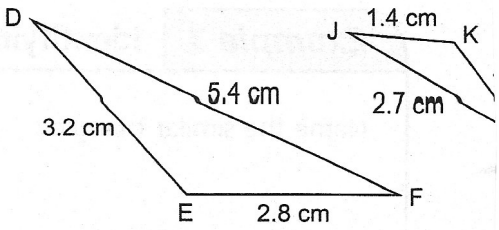
To name the triangles, order the letters so matching angles correspond.
△ABC ~ △_____

b) Find out if matching sides are proportional.
In △DEF, order the sides from shortest to longest:

_____

In △JKL, order the sides from shortest to longest:

_____



Find the scale factors of matching sides.

$\frac{\text{length of } \_\_\_}{\text{length of } \_\_\_} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

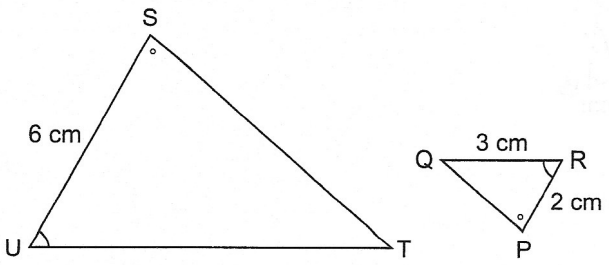
$\frac{\text{length of } \_\_\_}{\text{length of } \_\_\_} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

$\frac{\text{length of } \_\_\_}{\text{length of } \_\_\_} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

All scale factors are _____. So, the triangles _____.
The two longer sides meet at vertices: ____ and ____
The two shorter sides meet at vertices: ____ and ____
The longest and shortest sides meet at vertices: ____ and ____
So, △DEF ~ △_____

Example 2 Using Similar Triangles to Determine a Length

These two triangles are similar.
Find the length of TU.



Solution

List matching angles:

$\angle S = \angle P$ $\angle T = \angle Q$ $\angle U = \angle R$

So, $\triangle STU \sim \triangle PQR$

$\triangle STU$ is an enlargement of $\triangle PQR$.

Choose a pair of matching sides whose lengths are both known:

$SU = 6$ cm and $PR = 2$ cm

$$\text{Scale factor} = \frac{\text{length on enlargement}}{\text{length on original}}$$

$$= \frac{6 \text{ cm}}{2 \text{ cm}}$$

$$= 3$$

Consider the triangle with the unknown length as a reduction or enlargement of the other triangle.

The scale factor is 3.

Use the scale factor to find the length of TU.

TU and QR are matching sides.

Length of QR: 3 cm

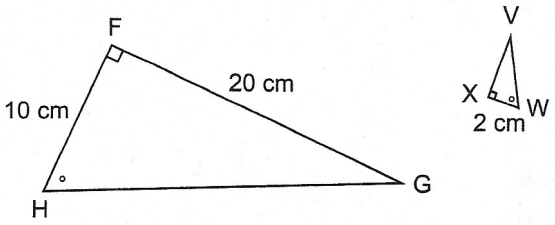
Scale factor: 3

Length of TU: $3 \times 3 \text{ cm} = 9 \text{ cm}$

So, TU has length 9 cm.

Check

1. These two triangles are similar.
Find the length of XV.



List matching angles:

$\angle F =$ _____ $\angle G =$ _____ $\angle H =$ _____

So, $\triangle FGH \sim \triangle$_____

_____ is a reduction of _____.

Choose a pair of matching sides whose lengths are both known:

_____

Scale factor $= \frac{\text{length on reduction}}{\text{length on original}}$

$= \frac{\quad}{\quad}$

$=$ _____

The scale factor is _____.

Use the scale factor to find the length of XV.

XV and FG are matching sides.

Length of FG: _____

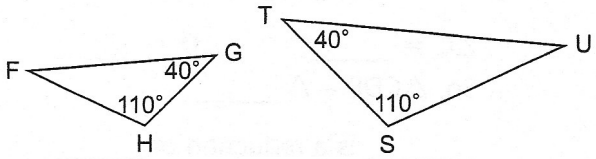
Scale factor: _____

Length of XV: _____

So, XV has length _____.

Practice

1. In each diagram, name two similar triangles.



a) Two angles in each triangle are given.
The measure of the third angle
in each triangle is: 180° − _____

List matching angles:
∠F = _____ = _____ ∠G = _____ = _____ ∠H = _____ = _____
Matching angles _____ equal, so, the triangles _____ similar.
To name the triangles, order the letters so matching angles correspond. △FGH ~ △_____

b) Find out if matching sides are proportional.
In △JKL, order the sides from shortest to longest: _____
In △QRS, order the sides from shortest to longest: _____
Find the scale factors of matching sides.

$\frac{\text{length of } \_\_\_}{\text{length of } \_\_\_} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

$\frac{\text{length of } \_\_\_}{\text{length of } \_\_\_} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

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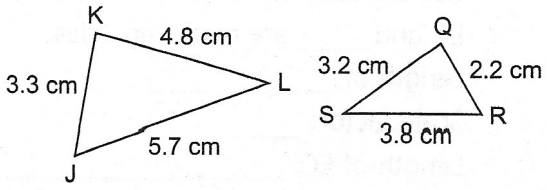
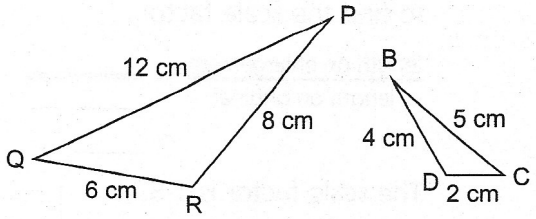


All scale factors are _____. So, the triangles _____.
The longest and shortest sides meet at vertices: ____ and ____
The two shorter sides meet at vertices: ____ and ____
The two longer sides meet at vertices: ____ and ____
So, △JKL ~ △_____

2. Are these two triangles similar?

In △PQR, order the sides from shortest to longest:
_____

In △BCD, order the sides from shortest to longest:
_____



Find the scale factors of matching sides.

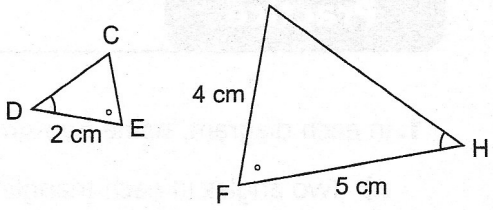
$\frac{\text{length of } \_\_\_}{\text{length of } \_\_\_} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

$\frac{\text{length of } \_\_\_}{\text{length of } \_\_\_} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

$\frac{\text{length of } \_\_\_}{\text{length of } \_\_\_} = \frac{\_\_\_}{\_\_\_} = \_\_\_$

All scale factors are _____. So, the triangles _____.

3. These two triangles are similar.
Find the length of EC.



List matching angles:

$\angle C =$ _____ $\angle D =$ _____ $\angle E =$ _____

So, $\triangle CDE \sim \triangle$_____

_____ is a reduction of _____.

Choose a pair of matching sides whose lengths are both known:

_____

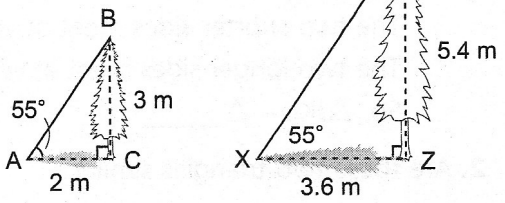
$$\text{Scale factor} = \frac{\text{length on reduction}}{\text{length on original}}$$

$$= \frac{\_\_\_\_}{\_\_\_\_}$$

$$= \_\_\_\_$$

The scale factor is _____.
Use the scale factor to find the length of EC.
EC and _____ are matching sides.
Length of _____: _____
Scale factor: _____
Length of EC: _____
So, EC has length _____.

4. At a certain time of day, two trees cast shadows.
Find the height of the taller tree.



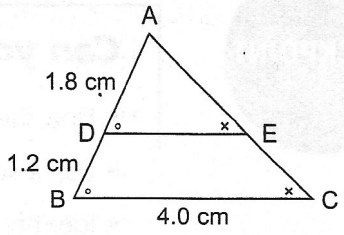
Matching angles are _____.
So, $\triangle ABC \sim \triangle$_____
$\triangle XYZ$ is an _____ of $\triangle ABC$.
Use sides _____
to find the scale factor.

$$\frac{\text{length on enlargement}}{\text{length on original}} = \frac{\_\_\_\_}{\_\_\_\_}$$

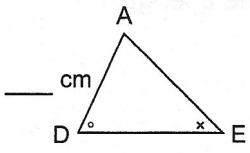
$$= \_\_\_\_$$

The scale factor is 1.8.
Use the scale factor to find the height of the taller tree, YZ.
BC and YZ are matching sides.
Length of BC: _____ Scale factor: _____
Length of YZ: _____
So, the height of the taller tree is _____.

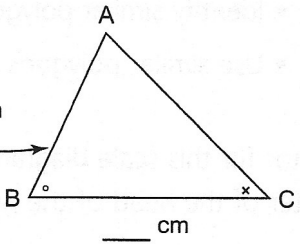
5. The two triangles in this diagram are similar.
Find the length of DE.



To better see the individual triangles, we draw the triangles separately.



1.8 cm + 1.2 cm = ____ cm



$\angle A =$ _____ $\angle B =$ _____ $\angle C =$ _____

So, $\triangle ABC \sim \triangle$_____

_____ is a reduction of _____.

Choose a pair of matching sides whose lengths are both known:

_____

Scale factor $= \frac{\text{length on reduction}}{\text{length on original}}$

$= \frac{\_\_\_\_}{\_\_\_\_}$

$=$ _____

The scale factor is _____.

Use the scale factor to find the length of DE.

_____ and _____ are matching sides.

Length of _____: _____

Scale factor: _____

Length of DE: _____

So, DE has length _____.