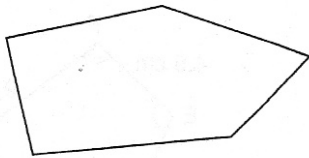


## 7.3 Skill Builder

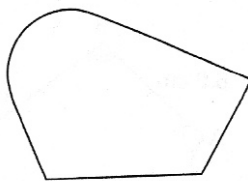
### Polygons

A **polygon** is a closed shape with straight sides.  
Exactly 2 sides meet at a vertex.

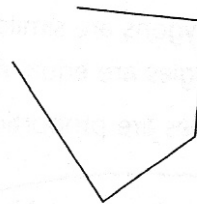
This shape is a polygon.



These shapes are **non-polygons**.



This shape has a curved side.

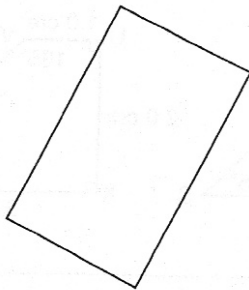


This shape is not closed.

### Check

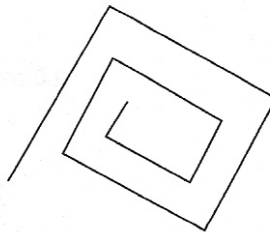
1. Is each shape a polygon or a non-polygon?

a)



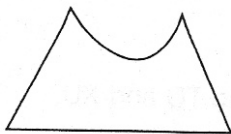
\_\_\_\_\_

b)



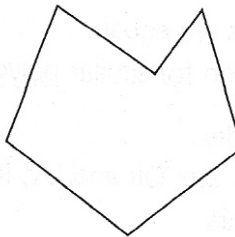
\_\_\_\_\_

c)



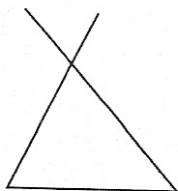
\_\_\_\_\_

d)



\_\_\_\_\_

e)



\_\_\_\_\_

f)



\_\_\_\_\_

## 7.3 Similar Polygons

**FOCUS** Recognize similar polygons, then use their properties to solve problems.

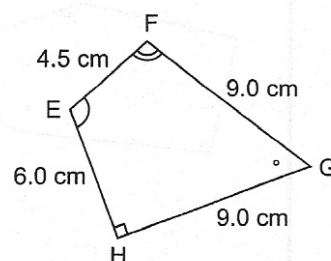
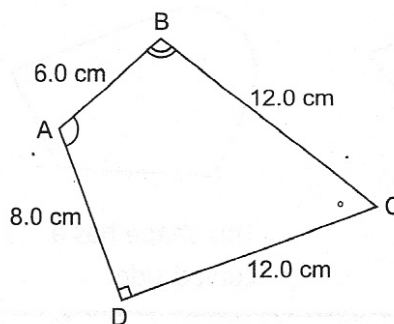
When one polygon is an enlargement or reduction of another polygon, we say the polygons are **similar**.

Similar polygons have the same shape, but not necessarily the same size.

When two polygons are similar:

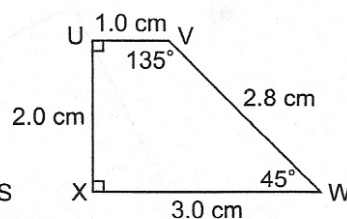
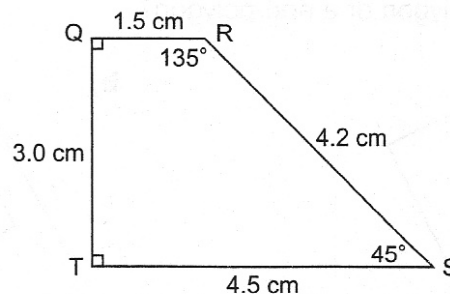
- matching angles are equal **AND**
- matching sides are proportional

*When all pairs of matching sides have the same scale factor, we say matching sides are **proportional**.*



### Example 1 Identifying Similar Polygons

Are these quadrilaterals similar? Explain.



### Solution

Check matching angles:  $\angle Q = \angle U = 90^\circ$      $\angle R = \angle V = 135^\circ$   
 $\angle S = \angle W = 45^\circ$      $\angle T = \angle X = 90^\circ$

All matching angles are equal.

So, the first condition for similar polygons is met.

Check matching sides.

The matching sides are: QR and UV, RS and VW, ST and WX, and TQ and XU.

Find the scale factors.

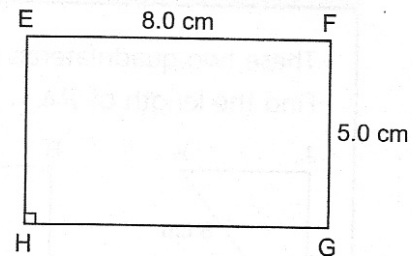
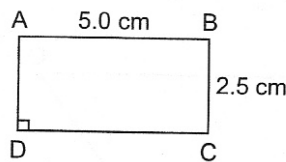
$$\begin{aligned} \frac{\text{length of QR}}{\text{length of UV}} &= \frac{1.5 \text{ cm}}{1.0 \text{ cm}} & \frac{\text{length of RS}}{\text{length of VW}} &= \frac{4.2 \text{ cm}}{2.8 \text{ cm}} \\ &= 1.5 & &= 1.5 \\ \frac{\text{length of ST}}{\text{length of WX}} &= \frac{4.5 \text{ cm}}{3.0 \text{ cm}} & \frac{\text{length of TQ}}{\text{length of XU}} &= \frac{3.0 \text{ cm}}{2.0 \text{ cm}} \\ &= 1.5 & &= 1.5 \end{aligned}$$

All scale factors are equal, so matching sides are proportional.

Since matching angles are equal and matching sides are proportional, the quadrilaterals are similar.

## Check

1. Are these rectangles similar?



Check matching angles.

The measure of each angle in a rectangle is \_\_\_\_\_.

So, matching angles are \_\_\_\_\_.

Check matching sides.

The matching sides are: \_\_\_\_\_ and \_\_\_\_\_, and \_\_\_\_\_ and \_\_\_\_\_.

Find the scale factors.

$$\frac{\text{length of } \underline{\hspace{1cm}}}{\text{length of } \underline{\hspace{1cm}}} = \underline{\hspace{1cm}} \quad \frac{\text{length of } \underline{\hspace{1cm}}}{\text{length of } \underline{\hspace{1cm}}} = \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} = \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

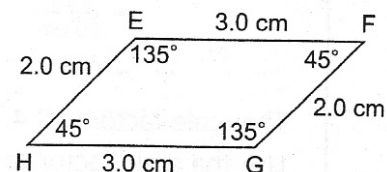
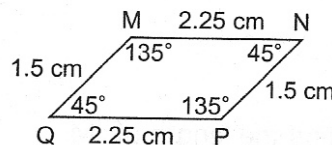
Since opposite sides of a rectangle are equal, check only one pair of matching lengths and one pair of matching widths.

The scale factors \_\_\_\_\_ equal.

So, the sides \_\_\_\_\_ proportional.

The rectangles \_\_\_\_\_ similar.

2. Are these parallelograms similar?



Check matching angles.  $\angle M = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

$\angle N = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

\_\_\_\_\_

\_\_\_\_\_

All matching angles \_\_\_\_\_ equal.

Check matching sides.

The matching sides are: \_\_\_\_\_ and \_\_\_\_\_, and \_\_\_\_\_ and \_\_\_\_\_.

Find the scale factors.

$$\frac{\text{length of } \underline{\hspace{1cm}}}{\text{length of } \underline{\hspace{1cm}}} = \underline{\hspace{1cm}} \quad \frac{\text{length of } \underline{\hspace{1cm}}}{\text{length of } \underline{\hspace{1cm}}} = \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} = \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Since opposite sides of a parallelogram are equal, check only two pairs of matching sides.

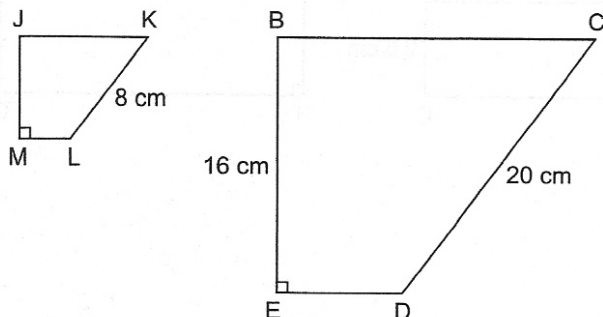
The scale factors \_\_\_\_\_ equal.

So, the sides \_\_\_\_\_ proportional.

The parallelograms \_\_\_\_\_ similar.

## Example 2 Determining Lengths in Similar Polygons

These two quadrilaterals are similar.  
Find the length of JM.



### Solution

Quadrilateral JKLM is a reduction of quadrilateral BCDE.

To find the scale factor of the reduction, choose a pair of matching sides whose lengths are both known:  
CD = 20 cm and KL = 8 cm

$$\begin{aligned}\text{Scale factor} &= \frac{\text{length on reduction}}{\text{length on original}} \\ &= \frac{8 \text{ cm}}{20 \text{ cm}} \\ &= 0.4\end{aligned}$$

The scale factor is 0.4.

Use the scale factor to find the length of JM.

JM and BE are matching sides.

Length of BE: 16 cm

Scale factor: 0.4

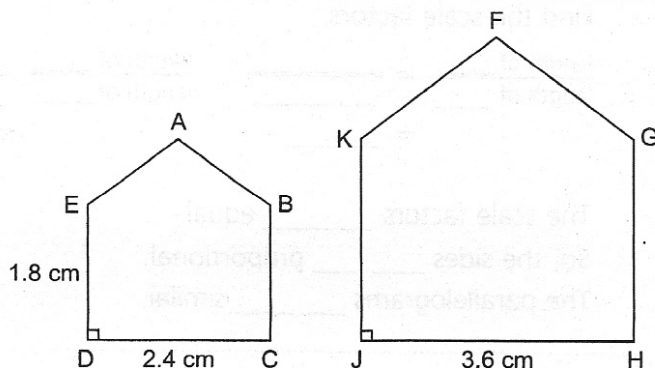
Length of JM:  $0.4 \times 16 \text{ cm} = 6.4 \text{ cm}$

So, JM has length 6.4 cm.

Consider the polygon with the unknown length as a reduction or enlargement of the other polygon.

### Check

- These two polygons are similar.  
Find the length of JK.



Polygon FGHJK is an enlargement of polygon ABCDE.

To find the scale factor, choose a pair of matching sides whose lengths are both known:

$$\text{Scale factor} = \frac{\text{length on enlargement}}{\text{length on original}}$$

$$= \frac{\quad}{\quad}$$

$$= \frac{\quad}{\quad}$$

The scale factor is \_\_\_\_\_.

Use the scale factor to find the length of JK.

JK and DE are matching sides.

Length of DE: \_\_\_\_\_

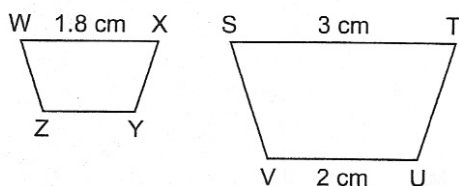
Scale factor: \_\_\_\_\_

Length of JK: \_\_\_\_\_

So, JK has length \_\_\_\_\_.

2. These two polygons are similar.

Find the length of YZ.



Polygon WXYZ is a \_\_\_\_\_ of polygon STUV.

To find the scale factor, choose a pair of matching sides whose lengths are both known:

$$\text{Scale factor} = \frac{\text{length on } \quad}{\text{length on original}}$$

$$= \frac{\quad}{\quad}$$

$$= \frac{\quad}{\quad}$$

The scale factor is \_\_\_\_\_.

Use the scale factor to find the length of YZ.

UV and YZ are matching sides.

Length of UV: \_\_\_\_\_

Scale factor: \_\_\_\_\_

Length of YZ: \_\_\_\_\_

So, YZ has length \_\_\_\_\_.



## Practice

1. Are these quadrilaterals similar?

Check matching angles.  $\angle A = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

$\angle B = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

$\underline{\hspace{1cm}}$   
 $\underline{\hspace{1cm}}$

All matching angles  $\underline{\hspace{1cm}}$  equal.

Check matching sides.

The matching sides are: AB and  $\underline{\hspace{1cm}}$ , and BC and  $\underline{\hspace{1cm}}$ .

Find the scale factors.

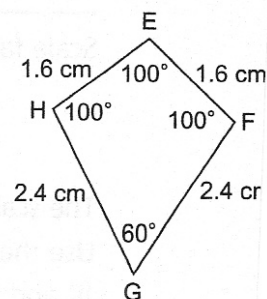
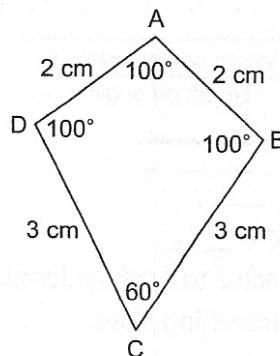
length of  $\underline{\hspace{1cm}}$  =  $\underline{\hspace{1cm}}$   
length of  $\underline{\hspace{1cm}}$  =  $\underline{\hspace{1cm}}$   
=  $\underline{\hspace{1cm}}$

length of  $\underline{\hspace{1cm}}$  =  $\underline{\hspace{1cm}}$   
length of  $\underline{\hspace{1cm}}$  =  $\underline{\hspace{1cm}}$   
=  $\underline{\hspace{1cm}}$

The scale factors  $\underline{\hspace{1cm}}$  equal.

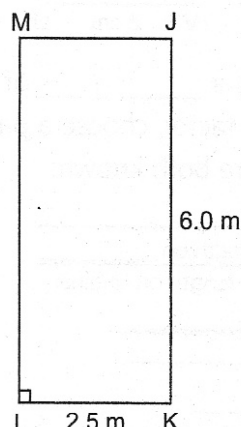
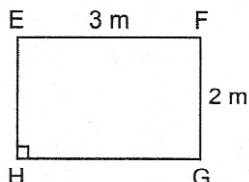
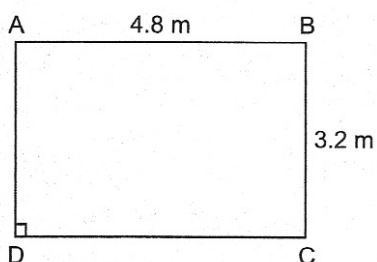
So, the sides  $\underline{\hspace{1cm}}$  proportional.

The quadrilaterals  $\underline{\hspace{1cm}}$  similar.



*Since adjacent sides of the kites are equal, check only two pairs of matching sides.*

2. Are any of these rectangles similar?



The measure of each angle in a rectangle is  $\underline{\hspace{1cm}}$ .

So, for any two rectangles, matching angles are  $\underline{\hspace{1cm}}$ .

Check matching lengths and widths in pairs of rectangles.

For rectangles ABCD and EFGH, the scale factors are:

length of  $\underline{\hspace{1cm}}$  =  $\underline{\hspace{1cm}}$   
length of  $\underline{\hspace{1cm}}$  =  $\underline{\hspace{1cm}}$   
=  $\underline{\hspace{1cm}}$

length of  $\underline{\hspace{1cm}}$  =  $\underline{\hspace{1cm}}$   
length of  $\underline{\hspace{1cm}}$  =  $\underline{\hspace{1cm}}$   
=  $\underline{\hspace{1cm}}$

The scale factors  $\underline{\hspace{1cm}}$  equal.

So, the sides  $\underline{\hspace{1cm}}$  proportional.

The rectangles  $\underline{\hspace{1cm}}$  similar.

For rectangles ABCD and JKLM, the scale factors are:

$$\frac{\text{length of } \underline{\hspace{2cm}}}{\text{length of } \underline{\hspace{2cm}}} = \frac{\underline{\hspace{2cm}}}{\underline{\hspace{2cm}}} \quad \frac{\text{length of } \underline{\hspace{2cm}}}{\text{length of } \underline{\hspace{2cm}}} = \frac{\underline{\hspace{2cm}}}{\underline{\hspace{2cm}}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

The scale factors                      equal.

So, the sides                      proportional.

The rectangles                      similar.

Is rectangle EFGH similar to rectangle JKLM?

Use what we know to find out.

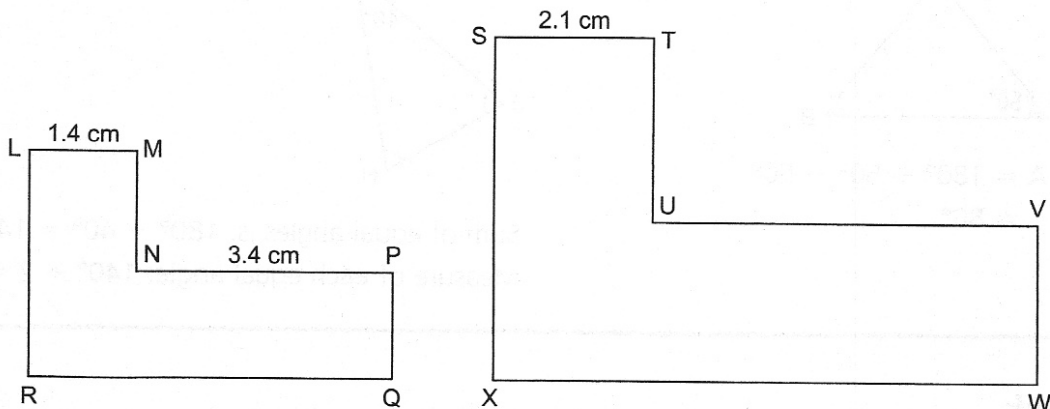
We know that rectangle ABCD                      to rectangle EFGH.

We know that rectangle ABCD                      to rectangle JKLM.

So, we know rectangle EFGH                      to rectangle JKLM.

**3.** These two polygons are similar.

Find the length of UV.



Polygon STUVWX is an enlargement of polygon LMNPQR.

To find the scale factor, choose a pair of matching sides whose lengths are both known:

$$\text{Scale factor} = \frac{\text{length on enlargement}}{\text{length on original}}$$

$$= \frac{\underline{\hspace{2cm}}}{\underline{\hspace{2cm}}}$$

$$= \underline{\hspace{2cm}}$$

The scale factor is                     .

Use the scale factor to find the length of UV.

UV and NP are matching sides.

Length of NP:                     

Scale factor:                     

Length of UV:                     

So, UV has length                     .