UNIT 7

Similarity and Transformations

What You'll Learn

- Draw and interpret scale diagrams.
- Apply properties of similar polygons.
- Identify and describe line symmetry and rotational symmetry.

Why It's Important

Similarity and scale diagrams are used by

- construction workers when they construct buildings and bridges
- motorists when they use maps to get around a city

Symmetry is used by

- interior designers when they arrange furniture and accessories in a room

Key Words

enlargement
reduction
scale diagram
scale factor
polygon
non-polygon
similar polygons
proportional
line symmetry
congruent
reflection
line of reflection
tessellation
rotation
rotational symmetry
order of rotation
angle of rotation symmetry
translation

7.1 Skill Builder

Converting Between Metric Units of Length

This table shows the relationships among some of the units of length.

1 m = 100 cm 1 m = 1000 mm
1 cm = 0.01 m 1 cm = 10 mm
1 mm = 0.001 m 1 mm = 0.1 cm

To convert 2.3 m to centimetres:
1 m = 100 cm
So, to convert metres to centimetres, multiply by 100.
2.3 m = 2.3(100 cm)
= 230 cm

To convert 255 cm to metres:
1 cm = 0.01 m
So, to convert centimetres to metres, multiply by 0.01.
255 cm = 255(0.01 m)
= 2.55 m

Check

1. Convert each measure to centimetres.

a) 7 m
1 m = _____ cm
So, 7 m = 7(_____)
= _____

b) 21 mm
1 mm = _____
So, 21 mm = _____
= _____

2. Convert each measure to metres.

a) 346 cm
1 cm = _____ m
So, 346 cm = 346(_____)
= _____

b) 1800 mm
1 mm = _____
So, 1800 mm = _____
= _____

3. Convert each measure to millimetres.

a) 6.5 cm
1 cm = _____ mm
So, 6.5 cm = 6.5(_____)
= _____

b) 3.8 m
1 m = _____
So, 3.8 m = _____
= _____

7.1 Scale Diagrams and Enlargements

FOCUS **Draw and interpret scale diagrams that represent enlargements.**

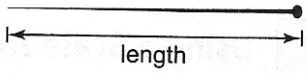
A diagram that is an **enlargement** or a **reduction** of another diagram is called a **scale diagram.** The **scale factor** is the relationship between the matching lengths on the two diagrams.

To find the scale factor of a scale diagram, we divide:

$$\frac{\text{length on scale diagram}}{\text{length on original diagram}}$$

Example 1 Using Matching Lengths to Determine the Scale Factor

Here is a scale diagram of a pin.
The actual length of the pin is 13 mm.
Find the scale factor of the diagram.



Solution

Measure the length of the pin in the diagram.
The length is 3.9 cm, or 39 mm.

The scale factor is: $\frac{\text{length on scale diagram}}{\text{length of pin}} = \frac{39 \text{ mm}}{13 \text{ mm}}$

$= 3$

The units of length must be the same.

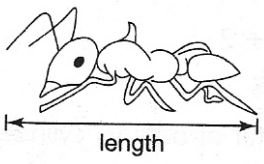
The scale factor is 3.

When the drawing is an enlargement, the scale factor is greater than 1.

Check

1. Find the scale factor for each scale diagram.

a) The actual length of the ant is 6 mm.



Measure the length of the ant in the diagram.
Length = _____ cm, or _____ mm

Scale factor = $\frac{\text{length on scale diagram}}{\text{length of ant}}$

$= \frac{\quad}{\quad}$

$=$ _____

The scale factor is _____.

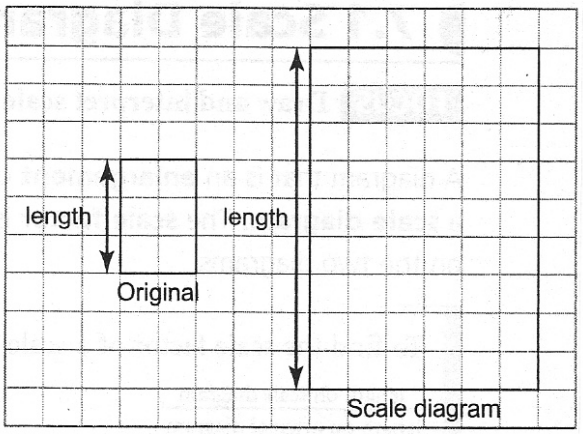
b) Length of rectangle in scale diagram: _____
Length of original rectangle: _____

Scale factor $= \frac{\text{length on scale diagram}}{\text{length on original diagram}}$

$= \frac{\quad}{\quad}$

$=$ _____

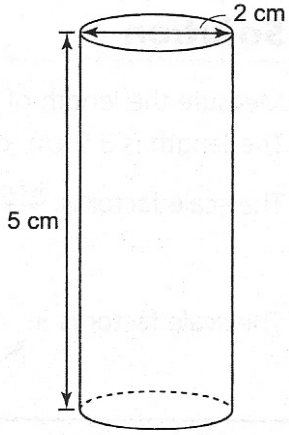
The scale factor is _____.



To find the dimensions of a scale diagram, multiply each length on the original diagram by the scale factor.

Example 2 Using a Scale Factor to Determine Dimensions

This cylinder is to be enlarged by a scale factor of $\frac{5}{2}$.
Find the dimensions of the enlargement.



Solution

Write the scale factor as a decimal.

$\frac{5}{2} = 5 \div 2$
$= 2.5$

To write a fraction as a decimal, divide the numerator by the denominator.

Diameter of original cylinder: 2 cm
Diameter of enlargement: 2.5×2 cm $= 5$ cm

Height of original cylinder: 5 cm
Height of enlargement: 2.5×5 cm $= 12.5$ cm

The enlargement has diameter 5 cm and height 12.5 cm.

Check

1. A photo has dimensions 10 cm by 15 cm.
Enlargements are to be made with each scale factor below.
Find the dimensions of each enlargement.

a) Scale factor 4

Length of original photo: _____
Length of enlargement: 4 × _____ = _____

The length of a rectangle is always the longer dimension.

Width of original photo: _____
Width of enlargement: 4 × _____ = _____

The enlargement has dimensions _____.

b) Scale factor $\frac{13}{4}$

Write the scale factor as a decimal.

_____

Length of original photo: _____
Length of enlargement: _____ = _____

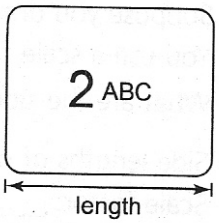
Width of original photo: _____
Width of enlargement: _____ = _____

The enlargement has dimensions _____.

Practice

1. Find the scale factor for each scale diagram.

a) The actual length of the cell phone button is 9 mm.
Measure the length of the button in the diagram.
Length = _____ cm, or _____ mm



Scale factor = $\frac{\text{length on scale diagram}}{\text{length of button}} = \frac{\_\_\_\_}{\_\_\_\_} =$ _____

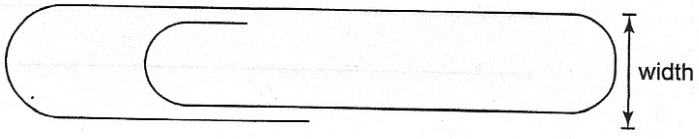
The scale factor is _____.

b) The actual width of the paperclip is 6 mm.

The width of the paperclip in the diagram is: Width = _____ cm, or _____ mm

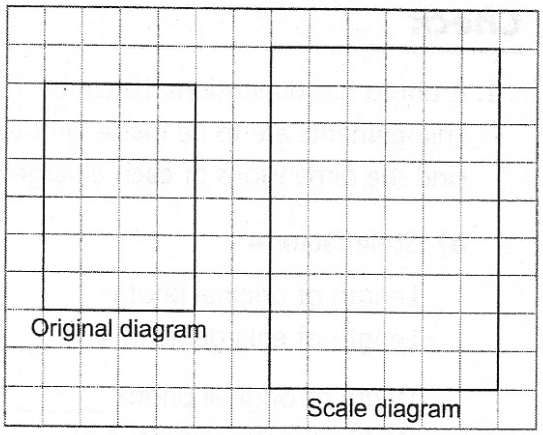
Scale factor = $\frac{\text{width on scale diagram}}{\text{width of paperclip}}$

$= \frac{\_\_\_\_}{\_\_\_\_} =$ _____



The scale factor is _____.

2. Find the scale factor for this scale diagram.



Original length: _____

Length on scale diagram: _____

Scale factor = $\frac{\text{length on scale diagram}}{\text{length on original diagram}}$

= _____

= _____

The scale factor is _____.

3. Enlargements of a photo are to be placed in different catalogues.
The original photo has side length 4 cm.
Find the side length for each enlargement of this photo.

a) Enlargement with scale factor 2.5

Side length of original photo: _____

Side length of enlargement: 2.5 × _____ = _____

The enlargement has side length _____.

b) Enlargement with scale factor $\frac{7}{4}$

Write the scale factor as a decimal:

_____

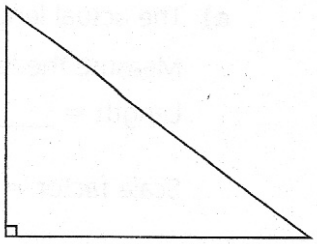
_____

Side length of original photo: _____

Side length of enlargement: _____ = _____

The enlargement has side length _____.

4. Suppose you draw a scale diagram of this triangle.
You use a scale factor of 2.75.
What are the side lengths of the enlargement?



Side lengths of original triangle: _____

Scale factor: _____

Side lengths of enlargement:

_____

_____

_____

_____