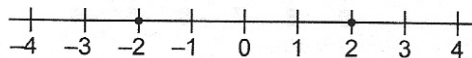# 6.5 Solving Linear Inequalities by Using Multiplication and Division

**FOCUS** Use multiplication and division to solve inequalities.

Consider the inequality $-2 < 2$.



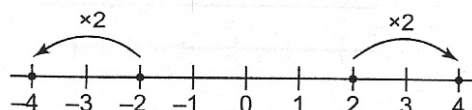What happens to an inequality when we multiply or divide each side by the same positive number?

$-2 < 2$      Multiply each side by 2.

Left side: $(-2)(2) = -4$

Right side: $2(2) = 4$
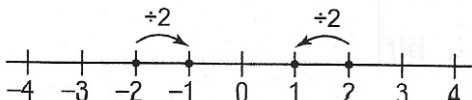
$-4 < 4$



The resulting inequality is still true.

$-2 < 2$      Divide each side by 2.

Left side: $\frac{-2}{2} = -1$

Right side: $\frac{2}{2} = 1$

$-1 < 1$



The resulting inequality is still true.

## Property of Inequalities

When each side of an inequality is multiplied or divided by the same positive number, the resulting inequality is still true.

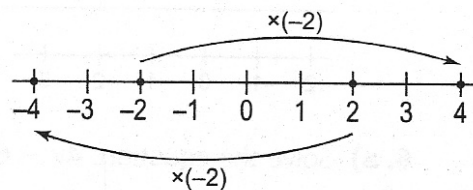What happens to an inequality when we multiply or divide each side by the same negative number?

$-2 < 2$      Multiply each side by $-2$.

Left side: $(-2)(-2) = 4$

Right side: $2(-2) = -4$

$4 > -4$



For the inequality to be true, the sign has to be reversed.

$-2 < 2$      Divide each side by $-2$.
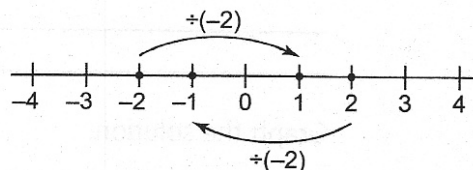
Left side: $\frac{-2}{-2} = 1$

Right side: $\frac{2}{-2} = -1$

$1 > -1$



For the inequality to be true, the sign has to be reversed.

## Property of Inequalities

When each side of an inequality is multiplied or divided by the same negative number, the inequality sign must be reversed for the inequality to remain true.

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**Example 1** | **Solving One-Step Inequalities**

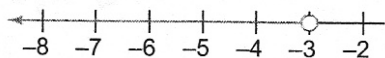Solve each inequality and graph the solution.

**a)** $4x < -12$      **b)** $-2c \geq 8$      **c)** $\frac{b}{2} \leq 3$      **d)** $\frac{v}{-3} > 4$

### Solution

**a)** $4x < -12$      Divide each side by 4.

$$\frac{4x}{4} < \frac{-12}{4}$$

> When you divide each side by the same positive number, do not reverse the inequality sign.

$x < -3$

The solution of $x < -3$ is all numbers less than $-3$.



**b)** $-2c \geq 8$      Divide each side by $-2$.

$$\frac{-2c}{-2} \leq \frac{8}{-2}$$

> When you divide each side by the same negative number, reverse the inequality sign.

$c \leq -4$

The solution of $c \leq -4$ is all numbers less than or equal to $-4$.
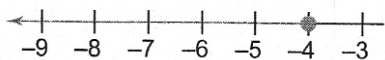


**c)**      $\frac{b}{2} \geq 3$      Multiply each side by 2.

$$2 \times \frac{b}{2} \geq 2(3)$$

> When you multiply each side by the same positive number, do not reverse the inequality sign.

$b \geq 6$

The solution of $b \geq 6$ is all numbers greater than or equal to 6.
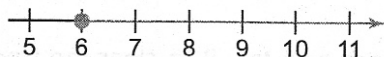
**d)** $\dfrac{v}{-3} > 4$        Multiply each side by $-3$.

*When you multiply each side by the same negative number, reverse the inequality sign.*

$$(-3)\left(\dfrac{v}{-3}\right) < (-3)(4)$$

$$v < -12$$

The solution of $v < -12$ is all numbers less than $-12$.
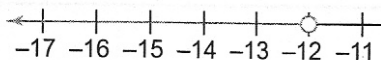


−17  −16  −15  −14  −13  −12  −11

## Check

**1.** State whether you would reverse the inequality sign to solve each inequality.

**a)** $-2m < 8$      **b)** $2m \le 8$      **c)** $\dfrac{y}{-2} > 3$

_____    _____    _____

**2.** Solve the inequalities in question 1. Graph each solution.

**a)** $-2m < 8$      **b)** $2m \le 8$      **c)** $\dfrac{y}{-2} > 3$

_____    _____    _____

_____    _____    _____



−5  −4  −3  −2  −1  0

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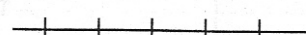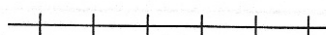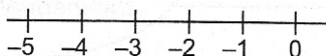
### Example 2 | Solving a Multi-Step Inequality

**a)** Solve the inequality: $1 - \dfrac{2}{3}x > 3$      **b)** Graph the solution.

## Solution

**a)** $\qquad 1 - \dfrac{2}{3}x > 3$        Subtract 1 from each side to isolate $x$.

$$1 - \dfrac{2}{3}x - 1 > 3 - 1$$

$$-\dfrac{2}{3}x > 2 \qquad$$ Multiply each side by $-3$ to clear the fraction. Reverse the inequality sign.

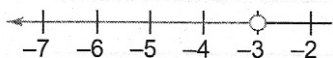$$(-3)\left(-\dfrac{2}{3}x\right) < (-3)(2)$$

$$2x < -6 \qquad$$ Divide each side by 2.

$$\dfrac{2x}{2} < \dfrac{-6}{2}$$

$$x < -3$$

**b)** The solution of $x < -3$ is all numbers less than $-3$.

$$\begin{array}{ccccccc} -7 & -6 & -5 & -4 & -3 & -2 \end{array}$$

## Check

**1.** Solve the inequality: $-\frac{2f}{5} < 4$

Graph the solution on the number line.

_____

_____

_____

_____

$$\begin{array}{cccccc} -11 & -10 & -9 & -8 & -7 & -6 \end{array}$$
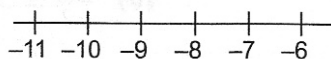
*If you multiply or divide by a negative number, remember to reverse the inequality sign.*

## Practice

**1. a)** Will the inequality sign change when you perform the indicated operation on each side of the inequality?

**i)** $3 > -2$; Multiply by 2 _____

**ii)** $4 \leq 8$; Divide by $-4$ _____

**iii)** $-5 < 1$; Multiply by $-5$ _____

**iv)** $1 > -4$; Divide by 1 _____

**b)** Perform each operation above. Write the resulting inequality.

**i)** _____  **ii)** _____

**iii)** _____  **iv)** _____

**2. a)** For the inequality $-2 < 6$, identify which of the following operations will reverse the inequality sign.

**i)** Multiply both sides by $-4$  **ii)** Divide both sides by 2

_____  _____

**b)** Perform each operation above. Write the resulting inequality.

**i)** _____  **ii)** _____

**3. a)** What operation do you have to do to solve each inequality?

**i)** $3x > 9$  **ii)** $-4p < -8$

_____  _____

**iii)** $-3y \leq 15$  **iv)** $\frac{q}{-2} \leq 5$

_____  _____

**b)** State whether you would reverse the inequality sign to solve each inequality in part a.
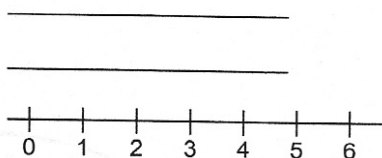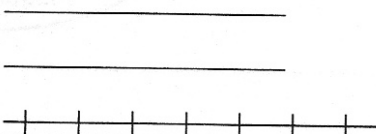
**i)** _____   **ii)** _____

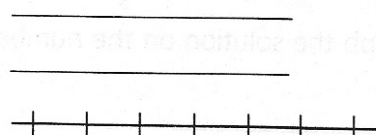**iii)** _____   **iv)** _____

**c)** Solve and graph each inequality.

**i)** $3x > 9$

_____

_____

$$\begin{array}{ccccccc} + & + & + & + & + & + & + \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

**iii)** $-3y \le 15$

_____

_____

$$+ \quad + \quad + \quad + \quad + \quad + \quad +$$

**ii)** $-4p < -8$

_____

_____

$$+ \quad + \quad + \quad + \quad + \quad + \quad +$$

**iv)** $\dfrac{q}{-2} \le 5$

_____

_____

$$+ \quad + \quad + \quad + \quad + \quad + \quad +$$
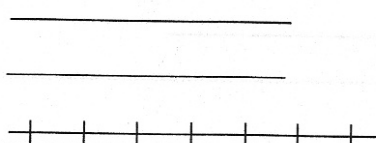
**4.** Solve each inequality. Graph the solution.

**a)** $3 - 2r \le 9$

_____

_____

_____

$$\begin{array}{cccccc} + & + & + & + & + & + \\ -5 & -4 & -3 & -2 & -1 & 0 \end{array}$$

**c)** $\dfrac{-s}{6} \ge 3$

_____

_____

_____

$$+ \quad + \quad + \quad + \quad + \quad +$$

**b)** $\dfrac{p}{5} + 2 > -3$

_____

_____

_____

$$+ \quad + \quad + \quad + \quad + \quad +$$

**d)** $\dfrac{5w}{8} - 1 < 4$

_____

_____

_____

_____

$$+ \quad + \quad + \quad + \quad + \quad +$$