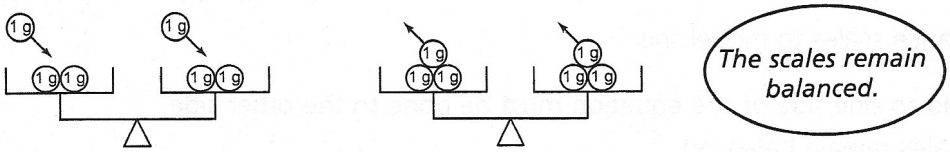
6.2 Skill Builder

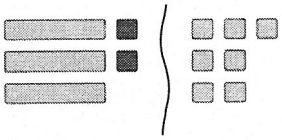
Solving Equations Using Models

We can use a balance-scales model to solve an equation.
Keep the scales balanced by doing the same operation on both sides.
For example, we can add or remove the same mass:

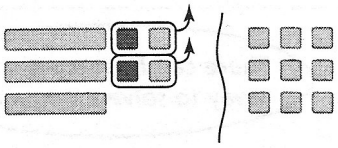


The scales remain balanced.

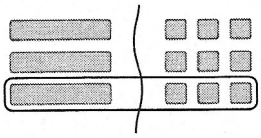
We can also use algebra tiles to solve an equation.
Rearrange the tiles so the variable tiles are on one side,
and the unit tiles are on the other side.
For example, to solve $3x - 2 = 7$:



Isolate the x-tiles.
Add 2 positive unit tiles to make zero pairs.



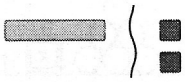
There are 3 x-tiles.
Arrange the unit tiles into 3 equal groups.



The solution is $x = 3$.

Check

1. Use algebra tiles to solve: $4m + 6 = -2$
Record your steps algebraically.



_____
_____
_____
_____

6.2 Solving Equations by Using Balance Strategies

FOCUS **Model a problem with a linear equation, use balance strategies to solve the equation pictorially, and record the process symbolically.**

To solve an equation, isolate the variable on one side of the equation.

We can use balance scales to model this.

Everything we do to one side of the equation must be done to the other side. This way, the scales remain balanced.

Example 1 Modelling Equations with Variables on Both Sides

a) Solve: $3x + 2 = 6 + x$

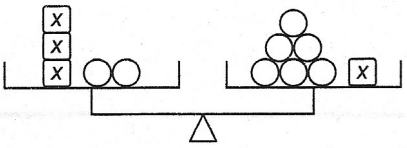
b) Verify the solution.

Solution

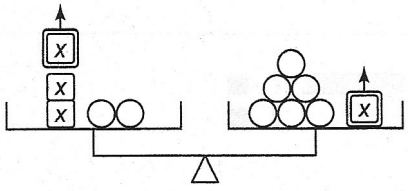
a) Isolate x to solve the equation.

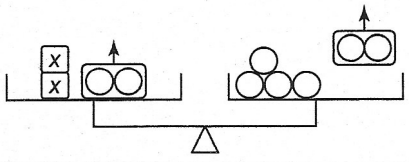
There can be more than one way to solve an equation.

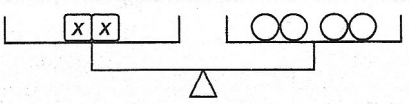
Pictorial Solution



Each ◯ has a mass of 1 g.







Algebraic Solution

$3x + 2 = 6 + x$

$3x + 2 - x = 6 + x - x$

$2x + 2 = 6$

$2x + 2 - 2 = 6 - 2$

$2x = 4$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

b) Check: Substitute $x = 2$ in each side of the equation.

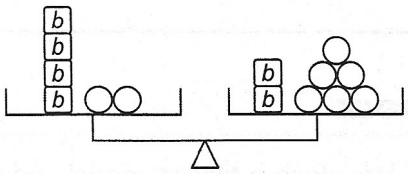
Left side $= 3x + 2$
$= 3(2) + 2$
$= 6 + 2$
$= 8$

Right side $= 6 + x$
$= 6 + 2$
$= 8$

Since the left side equals the right side, $x = 2$ is correct.

Check

1. Write the equation given by the picture.
Solve, and record your steps algebraically.



$4b + \_\_\_\_ = \_\_\_\_ + \_\_\_\_$

$4b + \_\_\_\_ - \_\_\_\_ = \_\_\_\_ + \_\_\_\_ - \_\_\_\_$

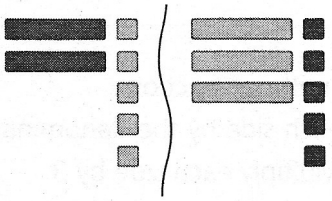
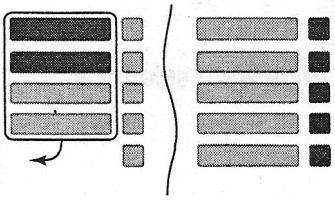
If we have an equation with negative terms, it is easier to use algebra tiles to model and solve the equation.

We add the same tiles to each side or subtract the same tiles from each side to keep the equation balanced.

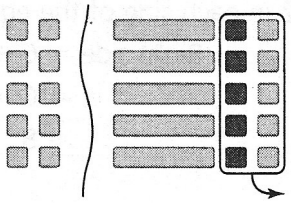
Example 2 Using Algebra Tiles to Solve an Equation

Solve: $-2n + 5 = 3n - 5$

Solution

	Algebra Tile Model	Algebraic Solution
		$-2n + 5 = 3n - 5$
Add 2 n-tiles to each side. Remove zero pairs.		$-2n + 5 + 2n = 3n - 5 + 2n$ $5 = 5n - 5$

Add five 1-tiles to each side.
Remove zero pairs.

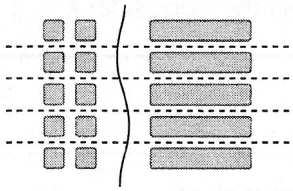


$5 + 5 = 5n - 5 + 5$

$10 = 5n$

You can solve with the variable on either side of the equal sign. The answer is the same.

Arrange the remaining tiles on each side into 5 groups. One n-tile is equal to 2.



$\frac{10}{5} = \frac{5n}{5}$

$2 = n$

Or, $n = 2$

Check

1. Use algebra tiles to model and solve the equation.
Record your work algebraically.

$-c + 5 = 2c - 4$

$-c + 5 - \_\_\_\_c = 2c - 4 - \_\_\_\_c$

$\_\_\_\_c + 5 = \_\_\_\_$

$\_\_\_\_c + 5 - \_\_\_\_ = \_\_\_\_ - \_\_\_\_$

$\_\_\_\_c = \_\_\_\_$

$c = \_\_\_\_$

Example 3 Solving Equations with Rational Coefficients

Solve the equation, then verify the solution.

$\frac{2a}{3} = 6$

Solution

Create an equivalent equation without fractions.
To clear the fraction, multiply each side by the denominator.

$\frac{2a}{3} = 6$ Multiply each side by 3.

$\frac{2a}{3} \times 3 = 6 \times 3$ $\frac{2a}{3} \times \frac{3}{1} = \frac{2a}{1} = 2a$

$2a = 18$ Divide each side by 2.

$\frac{2a}{2} = \frac{18}{2}$

$a = 9$

Check: Substitute $a = 9$ in $\frac{2a}{3} = 6$.

Left side $= \frac{2a}{3}$ Right side $= 6$

$= \frac{2(9)}{3}$

$= \frac{18}{3}$

$= 6$

Since the left side equals the right side, $a = 9$ is correct.

Check

1. Solve. Verify the solution.

a) $\frac{x}{4} = 5$

Clear the fraction. Multiply each side by the denominator, 4.

_____

_____

Check: Substitute $x =$ ____ in $\frac{x}{4} = 5$.

Left side = ____ Right side = ____

= ____

Since _____ $x =$ ____ is correct.

b) $\frac{x}{4} + \frac{7}{4} = \frac{5}{4}$

_____

_____

_____

_____

Check: Substitute $x =$ ____ in $\frac{x}{4} + \frac{7}{4} = \frac{5}{4}$.

Left side = _____ Right side = ____

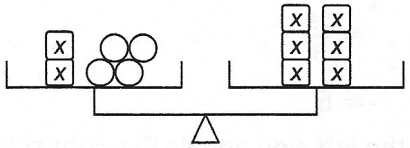
_____

_____

Since _____ $x =$ ____ is correct.

Practice

1. Write the equation represented by the picture. Solve, and record your steps algebraically.



2. Solve each equation.

a) $3w - 2 = w + 4$

$3w - 2$ ____ $= w + 4$ ____

b) $2 - x = -2 - 3x$

$2 - x$ ____ $= -2 - 3x$ ____

c) $y - 4 = -2 - y$

d) $2 - j = -8 + 4j$

3. Solve each equation. Verify the solution.

a) $\frac{t}{6} + 2 = 4$

$\frac{t}{6} + 2 -$ ____ $= 4 -$ ____

$\frac{t}{6} =$ ____

Left side $= \frac{t}{6} + 2$

$=$ ____

$=$ ____

$=$ ____

Right side = ____

$t =$ ____ is correct.

CHECKPOINT

Can you ...

- Model a problem with a linear equation, and solve the equation pictorially and symbolically?
- Model a problem with a linear equation, use balance strategies to solve the equation pictorially, and record the process symbolically?

6.1 **1.** For each equation, what is the first operation you would do to isolate the variable?

a) $3k = 9$

b) $m - 2 = 5$

c) $2x - 3 = 4$

d) $5 = 3y - 4$

2. For each statement, write then solve an equation to find the number. Verify the solution.

a) Two times a number is 10.

b) Three less than a number is 15.

3. Solve each equation.

a) $x + 7 = -2$

b) $4c = 20$

c) $4 = y - 2$

d) $\frac{m}{6} = 3$

4. Solve each equation. Verify the solution.

a) $3q - 1 = 17$

b) $2(3 + p) = -4$

b) $5 + \frac{w}{5} = 2$

Left side $= 5 + \frac{w}{5}$

_____ = _____

_____ = _____

_____ _____

_____ Right side = _____

$w =$ ____ is correct.

4. Jake tried to solve $4c - 3 = c + 3$ like this:

$4c - 3 - 3 = c + 3 - 3$

$4c = c + 0$

$4c - c = c - c + 0$

$3c = 0$

$c = 0$

a) Where did he make a mistake?

_____

b) Show the correct way to solve $4c - 3 = c + 3$.

Verify the solution.

$4c - 3 = c + 3$

_____

_____

_____

_____

_____

_____

Left side $= 4c - 3$

_____

_____

_____

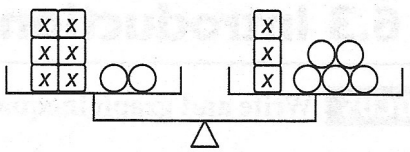
Right side $= c + 3$

_____

_____

Since the left side equals the right side, $c =$ ___ is correct.

6.2 **5.** Write the equation represented by the picture. Solve, and record your steps algebraically.



____ x + ____ = ____ x + ____

6. Solve each equation. Verify the solution.

a) $3a - 2 = a - 6$

$3a - 2 + 2 = a - 6 + 2$

Left side = $3a - 2$

=

Right side = $a - 6$

=

a = ___ is correct.

b) $4 + h = 1 - 2h$

$4 + h - 4 = 1 - 2h - 4$

Left side = $4 + h$

=

Right side = $1 - 2h$

=

h = ___ is correct.

c) $\frac{5a}{6} = 10$

Left side = $\frac{5a}{6}$

=

Right side =

a = ___ is correct.