

UNIT  
**6**

# Linear Equations and Inequalities

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## What You'll Learn

- Expand your understanding of solving equations.
- Model and solve problems using linear equations.
- Investigate the properties of inequalities.
- Explain and illustrate strategies to solve linear inequalities.

## Why It's Important

Linear equations and inequalities are used by

- nurses, home health aides, and medical assistants, to take temperatures and blood pressures, and set up equipment
- purchasing agents and buyers, to find the best merchandise at the lowest price for their employers, and stay aware of changes in the marketplace

## Key Words

inverse operations

variable

inequality

## 6.1 Skill Builder

### Order of Operations

We use this order of operations to evaluate expressions with more than one operation.

- B** Do the operations in **b**rackets first  
**E** Evaluate any **e**xponents  
**D**  
**M** Divide and **m**ultiply in order from left to right  
**A**  
**S** Add and **s**ubtract in order from left to right

$$\begin{aligned} & 7 - 8 \div 2 + (6 - 1) && \text{Evaluate brackets first: } (6 - 1) \\ = & 7 - 8 \div 2 + 5 && \text{Then divide: } 8 \div 2 \\ = & 7 - 4 + 5 && \text{Then add and subtract from left to right.} \\ = & 3 + 5 \\ = & 8 \end{aligned}$$

### Check

1. In each expression, circle what you will do first.

- |  |          |          |          |        |
|--|----------|----------|----------|--------|
| <b>a)</b> $-7 + 2 \times (-3)$           | Add      | Multiply |          |        |
| <b>b)</b> $3 \times (-10 \div 2) - (-4)$ | Multiply | Divide   | Subtract |        |
| <b>c)</b> $19 - 4 \times 3^2 \div 6$     | Subtract | Multiply | Power    | Divide |
| <b>d)</b> $-30 \div 5 - 10 \times 2$     | Divide   | Subtract | Multiply |        |

2. Evaluate.

- |                                    |   |   |
|------------------------------------|---|---|
| <b>a)</b> $-17 + 4 \times 3$       | <b>b)</b> $-16 \div 4 + 24 \div (-8)$                     | <b>c)</b> $3^2 + 4^2 \div 8 + (-5)$                                   |
| $= -17 + \underline{\hspace{1cm}}$ | $= \underline{\hspace{1cm}} + 24 \div (-8)$               | $= \underline{\hspace{1cm}} + \underline{\hspace{1cm}} \div 8 + (-5)$ |
| $= \underline{\hspace{1cm}}$       | $= \underline{\hspace{1cm}} + (\underline{\hspace{1cm}})$ | $= 9 + \underline{\hspace{1cm}} + (-5)$                               |
|                                    | $= \underline{\hspace{1cm}}$                              | $= \underline{\hspace{1cm}} + (-5)$                                   |
|                                    |   | $= \underline{\hspace{1cm}}$  |

## The Distributive Property

To multiply  $5 \times (3 + 4)$ , we can:

- Add  $3 + 4$  in the brackets, then multiply the sum by 5:

$$\begin{aligned} 5 \times (3 + 4) \\ = 5 \times 7 \\ = 35 \end{aligned}$$

OR

- Multiply each number in the brackets by 5, then add:

$$\begin{aligned} 5 \times (3 + 4) &= 5 \times 3 + 5 \times 4 \\ &= 15 + 20 \\ &= 35 \end{aligned}$$

We can use the distributive property to write this expression as a sum of terms:

$$7(a + b) = 7a + 7b$$

## Check

1. Expand.

a)  $3(b - 2)$

$$\begin{aligned} &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

b)  $6(2 - y)$

$$\begin{aligned} &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

## 6.1 Solving Equations by Using Inverse Operations

**FOCUS** Model a problem with a linear equation, and solve the equation pictorially and symbolically.

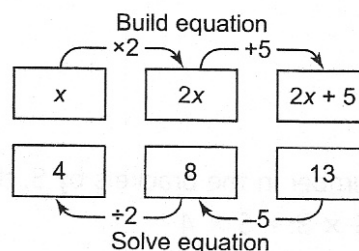
Look at the equation  $2x + 5 = 13$ .

How was it built?

Start with  $x$ . Multiply by 2, then add 5.

To solve the equation, "undo" the operations, in reverse order.

$$\begin{aligned} 2x + 5 &= 13 && \text{Subtract 5.} \\ 2x + 5 - 5 &= 13 - 5 \\ 2x &= 8 && \text{Divide by 2.} \\ x &= 4 \end{aligned}$$



Do the same operation to both sides of the equation to preserve the equality.

**Inverse operations** undo each other's results. For example: addition and subtraction are inverse operations.

### Example 1 Writing Then Solving One-Step Equations

Write then solve an equation to find each number. Verify the solution.

**a)** A number plus 5 is 20.

**b)** Four times a number is  $-32$ .

#### Solution

**a)** Let  $x$  represent the number. Then,  $x$  plus 5 is 20.

The equation is:  $x + 5 = 20$

To solve the equation, apply the inverse operations.

$$\begin{aligned} x + 5 &= 20 && \text{Undo the addition. Subtract 5 from each side.} \\ x + 5 - 5 &= 20 - 5 \\ x &= 15 \end{aligned}$$

To verify the solution, substitute  $x = 15$  into  $x + 5 = 20$ .

$15 + 5 = 20$ , so the solution is correct.

**b)** Let  $n$  represent the number. Then, 4 times  $n$  is  $-32$ .

The equation is:  $4n = -32$

To solve the equation, apply the inverse operations.

$$\begin{aligned} 4n &= -32 && \text{Undo the multiplication. Divide each side by 4.} \\ \frac{4n}{4} &= \frac{-32}{4} \\ n &= -8 \end{aligned}$$

To verify the solution, substitute  $n = -8$  into  $4n = -32$ .

$4(-8) = -32$ , so the solution is correct.

## Check

1. Let  $n$  represent a number. Two less than a number is 10. What is the number?

$$n - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$n - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$n = \underline{\hspace{2cm}}$$

Check:  $\underline{\hspace{2cm}}$

## Example 2 Solving a Two-Step Equation

Solve, then verify each equation.

a)  $3x + 4 = -5$

b)  $2(-2 + w) = 18$

### Solution

- a) Perform the inverse operations in reverse order.

$$3x + 4 = -5$$

Subtract 4 from each side.

$$3x + 4 - 4 = -5 - 4$$

$$3x = -9$$

Divide each side by 3.

$$\frac{3x}{3} = \frac{-9}{3}$$

$$x = -3$$

To verify the solution, substitute  $x = -3$  into  $3x + 4 = -5$ .

$$\text{Left side} = 3x + 4$$

$$\text{Right side} = -5$$

$$= 3(-3) + 4$$

$$= -9 + 4$$

$$= -5$$

Since the left side equals the right side,  $x = -3$  is correct.

b)  $2(-2 + w) = 18$

Use the distributive property to expand  $2(-2 + w)$ .

$$2(-2) + 2(w) = 18$$

$$-4 + 2w = 18$$

Add 4 to each side.

$$-4 + 2w + 4 = 18 + 4$$

$$2w = 22$$

Divide each side by 2.

$$\frac{2w}{2} = \frac{22}{2}$$

$$w = 11$$

To verify the solution, substitute  $w = 11$  into  $2(-2 + w) = 18$ .

$$\text{Left side} = 2(-2 + w)$$

$$\text{Right side} = 18$$

$$= 2(-2 + 11)$$

$$= 2(9)$$

$$= 18$$

Since the left side equals the right side,  $w = 11$  is correct.



## Check

1. What operations would you use to solve each equation?

a)  $-5h + 4 = 6$

First \_\_\_\_\_, then \_\_\_\_\_.

b)  $2 + 5p = -3$

First \_\_\_\_\_, then \_\_\_\_\_.

2. Solve, then verify the equation.

$$2(t - 1) = 12$$

$$2(\underline{\quad}) - 2(\underline{\quad}) = 12$$

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Use the distributive property  
to expand  $2(t - 1)$ .

Substitute  $t = \underline{\quad}$  into the equation.

$$\text{Left side} = 2(t - 1)$$

$$= 2(\underline{\quad})$$

$$= \underline{\quad}$$

$$= \underline{\quad}$$

$$\text{Right side} = \underline{\quad}$$

Since the left side equals the right side,  $t = \underline{\quad}$  is correct.

## Practice

1. Solve each equation.

a)  $z + 9 = 10$

\_\_\_\_\_  
\_\_\_\_\_

b)  $s - 4 = -12$

\_\_\_\_\_  
\_\_\_\_\_

c)  $6 + c = 2$

\_\_\_\_\_  
\_\_\_\_\_

d)  $5 = v - 2$

\_\_\_\_\_  
\_\_\_\_\_

2. For each statement, write then solve an equation to find the number.  
Verify the solution.

a) A number divided by 4 is  $-3$ .

$$\underline{\hspace{1cm}} = -3$$

\_\_\_\_\_

\_\_\_\_\_

Left side = \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Right side = \_\_\_\_\_

$n = \underline{\hspace{1cm}}$  is correct.

b) Three times a number is 15.

$$\underline{\hspace{1cm}} = 15$$

\_\_\_\_\_

\_\_\_\_\_

Left side = \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Right side = \_\_\_\_\_

$x = \underline{\hspace{1cm}}$  is correct.

3. Emma tried to solve the equation  $4x = 16$  by subtracting 4 from each side.  
Show the correct way to solve the equation.

$$4x = 16$$

\_\_\_\_\_

\_\_\_\_\_

4. Solve each equation. Verify the solution.

a)  $5k - 6 = 24$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Left side =  $5k - 6$   
 $= 5(\underline{\hspace{1cm}}) - 6$   
 $= \underline{\hspace{2cm}}$

Right side = \_\_\_\_\_  
 $k = \underline{\hspace{1cm}}$  is correct.

b)  $3 + 4y = -9$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Left side =  $3 + 4y$   
 $= \underline{\hspace{2cm}}$   
 $= \underline{\hspace{2cm}}$

Right side = \_\_\_\_\_  
 $y = \underline{\hspace{1cm}}$  is correct.

5. a) Tuyen tried to solve the equation  $3x - 6 = 15$  like this:

$$\frac{3x}{3} - 6 = \frac{15}{3}$$

$$x - 6 = 5$$

$$x - 6 + 6 = 5 + 6$$

$$x = 11$$

Where did she make a mistake?

\_\_\_\_\_

- b) Show the correct way to solve  $3x - 6 = 15$ .

Verify the solution.

Left side =  $3x - 6$

= \_\_\_\_\_

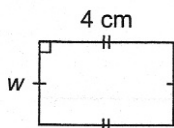
= \_\_\_\_\_

= \_\_\_\_\_

Right side = \_\_\_\_\_

Since the left side equals the right side,  $x = \underline{\hspace{1cm}}$  is correct.

6. A rectangle has length 4 cm and perimeter 12 cm.



*The perimeter is the sum of all the sides.*

- a) Write an equation that can be used to determine the width of the rectangle.

\_\_\_\_\_ = 12

- b) Solve the equation.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

The width is \_\_\_\_\_ cm.

- c) Verify the solution.

Left side = \_\_\_\_\_

Right side = \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_