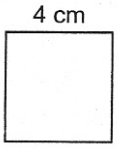
5.6 Skill Builder

Multiplying Monomials

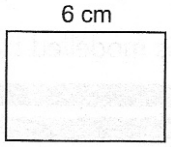
The area of this square is:



$4 \times 4 = 16$

Area $= 16\ cm^2$

The area of this rectangle is:

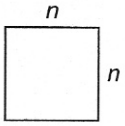


$4 \times 6 = 24$

Area $= 24\ cm^2$

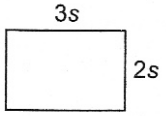
We can use the models above to help us multiply 2 monomials.

The area of this square is:



$n \times n = n^2$

The area of this rectangle is:



$$\begin{aligned} 3s \times 2s &= 3 \times s \times 2 \times s \\ &= 3 \times 2 \times s \times s \\ &= 6 \times s^2 \\ &= 6s^2 \end{aligned}$$

When one or both of the monomials is negative, we cannot use an area model.

We multiply using the rules for multiplying integers.

$$\begin{aligned} 4v \times (-2v) &= 4 \times v \times (-2) \times v \\ &= 4 \times (-2) \times v \times v \\ &= -8 \times v^2 \\ &= -8v^2 \end{aligned}$$

4 and −2 have different signs, so their product is negative.

Check

1. Multiply.

a) $b \times b =$ _____

b) $c \times (-c) =$ _____

c) $(-f) \times (-f) =$ _____

d) $(-g) \times g =$ _____

2. Multiply.

a) $5r \times 6r = 5 \times r \times 6 \times r$

$= 5 \times 6 \times r \times r$

$=$ _____

$=$ _____

b) $(-2d) \times 8d = (-2) \times d \times 8 \times d$

$= (-2) \times 8 \times d \times d$

$=$ _____

$=$ _____

c) $4a \times (-7a) =$ _____

$=$ _____

$=$ _____

$=$ _____

d) $(-5v) \times (-9v) = (-5) \times v \times (-9) \times v$

$=$ _____

$=$ _____

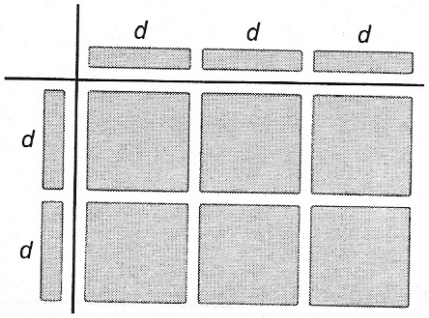
$=$ _____

5.6 Multiplying and Dividing a Polynomial by a Monomial

FOCUS **Use different strategies to multiply and divide a polynomial by a monomial.**

To multiply $2d(3d)$ with algebra tiles:

Draw 2 adjacent sides of a rectangle.
Position ▭ tiles to show side lengths $2d$ and $3d$.



$d \times d = d^2$, so use a d^2-tile.

Then fill the rectangle with tiles.

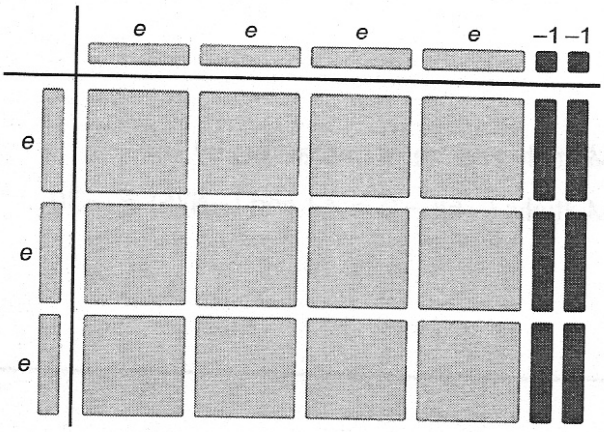
We used 6 d^2-tiles to fill the rectangle. So, $2d(3d) = 6d^2$

Example 1 Multiplying a Binomial by a Monomial

Find the product: $3e(4e - 2)$

Solution

$3e(4e - 2)$
Draw 2 adjacent sides of a rectangle.
Position tiles to show side lengths $3e$ and $4e - 2$.



When two tiles have the same colour, use a positive tile in the rectangle.

When two tiles have different colours, use a negative tile in the rectangle.

Fill the rectangle with tiles.
We used 12 e^2-tiles and 6 $-e$-tiles to fill the rectangle.
So, $3e(4e - 2) = 12e^2 - 6e$

Check

1. Sketch algebra tiles to multiply. Write the product each time.

a) $2f(4f)$

b) $3m(-2m + 4)$

Number of f^2-tiles used: ____

So, $2f(4f) =$ ____f^2

$3m(-2m + 4) =$ _____

Example 2 Multiplying a Binomial by a Monomial Symbolically

Find each product:

a) $3x(9x - 4)$ **b)** $-6x(-7x + 5)$

Solution

a) $3x(9x - 4)$

Use the distributive property.

Multiply each term in brackets by $3x$.

$= (3x)(9x) + (3x)(-4)$

Multiply: $3 \times 9 = 27$ and $3(-4) = -12$

$= 27x^2 + (-12x)$

$= 27x^2 - 12x$

b) $-6x(-7x + 5)$

Multiply each term in brackets by $-6x$.

$= (-6x)(-7x) + (-6x)(5)$

Multiply: $(-6)(-7) = 42$ and $(-6)(5) = -30$

$= 42x^2 + (-30x)$

$= 42x^2 - 30x$

Check

1. Multiply.

a) $7x(4x + 5)$

$= (7x)(4x) + (7x)(5)$

$= \_\_\_\_x^2 + \_\_\_\_x$

Multiply each term in brackets by $7x$.
Multiply: $7 \times 4 =$ ____ and $7 \times 5 =$ ____

b) $s(-3s + 4)$

$= (s)(-3s) + (s)(4)$

$=$ _____

Multiply each term in brackets by s.
Multiply: $1 \times (-3) =$ ____ and $1 \times 4 =$ ____

c) $-9r(4r - 5)$

$=$ _____

$=$ _____

To divide a polynomial by a monomial, we use what we already know about division.

To divide: $\frac{6a^2}{3a}$

We write the fraction as a product of 2 fractions, then simplify each fraction.

$\frac{6a^2}{3a} = \frac{6}{3} \times \frac{a^2}{a}$

$= 2 \times \frac{a \times \cancel{a}^{1}}{\cancel{a}_{1}}$

$= 2 \times a$

$= 2a$

a is a common factor of the numerator and the denominator.

$2a$ is the *quotient* of $\frac{6a^2}{3a}$.

Example 3 Dividing a Binomial by a Monomial

Find the quotient: $\frac{-8s^2 + 6s}{-2s}$

Solution

$\frac{-8s^2 + 6s}{-2s}$

Write as the sum of 2 fractions each with denominator $-2s$.

$= \frac{-8s^2}{-2s} + \frac{6s}{-2s}$

Simplify the fractions.

$= \frac{-8}{-2} \times \frac{s^2}{s} + \frac{6}{-2} \times \frac{s}{s}$

$= 4 \times s + (-3) \times 1$

$= 4s - 3$

A variable divided by itself is 1.

Check

1. Divide.

a) $\frac{12a^2}{-6a}$

Write as a product of 2 fractions.

$= \_\_\_ \times \frac{a^2}{a}$

Simplify each fraction.

$= \_\_\_ \times \frac{a \times a^1}{a \; 1}$

$= \_\_\_\_\_$

$= \_\_\_\_\_$

b) $\frac{9b^2 + 3b}{3b}$

$= \frac{\_\_\_\_}{3b} + \frac{\_\_\_\_}{3b}$

$= \_\_\_\_\_$

$= \_\_\_\_\_$

$= \_\_\_\_\_$

c) $\frac{-14c^2 + 21c}{-7c}$

$= \_\_\_\_\_$

$= \_\_\_\_\_$

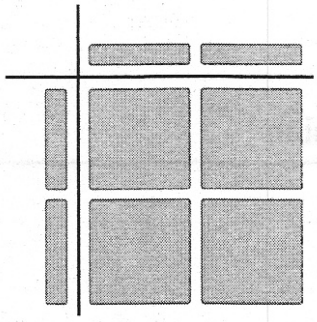
$= \_\_\_\_\_$

$= \_\_\_\_\_$

Practice

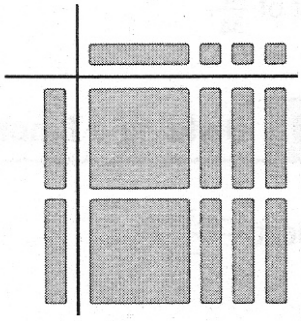
1. Write the multiplication sentence modelled by each set of tiles.

a)



_____

b)



_____

2. Sketch algebra tiles to multiply. Write the product each time.

a) $2s(s + 4) =$ _____

b) $t(-2t + 3) =$ _____

3. Multiply.

a) $4r(5r - 1)$

$= (4r)(\_\_\_) + (4r)(\_\_\_)$

$= \_\_\_r^2 + (\_\_\_r)$

$=$ _____

b) $7s(-3s + 6)$

$=$ _____

$=$ _____

c) $-6t(t - 3)$

$=$ _____

$=$ _____

d) $-8u(-6u + 7)$

$=$ _____

$=$ _____

4. Divide.

a) $\frac{12v^2}{4v}$

$= \_\_\_ \times \frac{v^2}{v}$

$= \_\_\_ \times \frac{v \times v^1}{v_1}$

$= \_\_\_ \times v$

$= \_\_\_$

b) $\frac{15w^2}{-3w}$

$=$ _____

$=$ _____

$=$ _____

c) $\frac{-28x^2}{-7x}$

$=$ _____

$=$ _____

$=$ _____

5. Divide.

a) $\frac{18y^2 + 12y}{2y}$

$= \frac{\_\_\_}{2y} + \frac{\_\_\_}{2y}$

$=$ _____

$=$ _____

$=$ _____

b) $\frac{-32z^2 + 24z}{-8z}$

$=$ _____

$=$ _____

$=$ _____

$=$ _____

c) $\frac{15n^2 + 21n}{-3n}$

$=$ _____

$=$ _____

$=$ _____

$=$ _____