

3.4 Skill Builder

Writing a Fraction in Simplest Form

A fraction is in simplest form when the only common factor of the numerator and denominator is 1. For example, $\frac{5}{6}$ is in simplest form.

Writing a Fraction in Simplest Form

Look for common factors of the numerator and denominator.

Divide the numerator and denominator by common factors until you cannot go any further.

Write $\frac{24}{30}$ in simplest form.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Divide the numerator and the denominator by 6.

$$\frac{24}{30} \xrightarrow{\div 6} \frac{4}{5}$$

$\frac{4}{5}$ is the simplest form of $\frac{24}{30}$.

Check

1. Write each fraction in simplest form.

a) $\frac{10}{15} \xrightarrow{\div 5} \frac{\quad}{\quad}$

Divide the numerator and the denominator by 5.

b) $\frac{14}{20} \xrightarrow{\quad} \frac{\quad}{\quad}$

Divide the numerator and the denominator by ____.

c) $\frac{8}{12} \xrightarrow{\quad} \frac{\quad}{\quad}$

Divide the numerator and the denominator by ____.

d) $\frac{12}{18} \xrightarrow{\quad} \frac{\quad}{\quad}$

Divide the numerator and the denominator by ____.

Multiplying Proper Fractions

When multiplying fractions, we multiply the numerators, and we multiply the denominators.

$$\frac{2}{5} \times \frac{3}{8} = \frac{2 \times 3}{5 \times 8}$$

$$= \frac{6}{40}, \text{ or } \frac{3}{20}$$

To simplify, look for common factors *before* multiplying.

$$\frac{5}{12} \times \frac{8}{15} = \frac{5 \times 8}{12 \times 15}$$

$$= \frac{5^1 \times 8^2}{12^3 \times 15^3}$$

$$= \frac{1 \times 2}{3 \times 3}$$

$$= \frac{2}{9}$$

A common factor of 5 and 15 is 5.

A common factor of 8 and 12 is 4.

$$5 \div 5 = 1$$

$$8 \div 4 = 2$$

$$12 \div 4 = 3$$

$$15 \div 5 = 3$$

Check

1. Find each product.

a) $\frac{3}{4} \times \frac{2}{5}$

Multiply the numerators and multiply the denominators.

$$= \frac{3 \times 2}{4 \times 5}$$

A common factor of 2 and 4 is ____.

$$= \frac{3 \times \cancel{2}}{\cancel{4} \times 5}$$

$$= \frac{\cancel{2} \times \cancel{2}}{\cancel{4} \times 5} = \frac{1 \times 1}{2 \times 5} = \frac{1}{10}$$

b) $\frac{9}{14} \times \frac{7}{3}$

Multiply the numerators and multiply the denominators.

$$= \frac{9 \times 7}{14 \times 3}$$

A common factor of 9 and 3 is ____.

A common factor of 7 and 14 is ____.

$$= \frac{9 \times 7}{14 \times 3}$$

$$= \frac{\cancel{9} \times \cancel{7}}{\cancel{14} \times \cancel{3}} = \frac{1 \times 1}{2 \times 1} = \frac{1}{2}$$

2. Multiply.

a) $\frac{6}{7} \times \frac{3}{4} = \frac{\cancel{6} \times 3}{7 \times \cancel{4}} = \frac{1 \times 3}{7 \times 1} = \frac{3}{7}$

b) $\frac{4}{5} \times \frac{15}{14} = \frac{4 \times \cancel{15}}{\cancel{5} \times 14} = \frac{1 \times 3}{1 \times 7} = \frac{3}{7}$

c) $\frac{12}{5} \times \frac{5}{18} = \frac{\cancel{12} \times \cancel{5}}{5 \times \cancel{18}} = \frac{1 \times 1}{1 \times 3} = \frac{1}{3}$

$$= \frac{6 \times 3}{7 \times 4}$$

$$= \frac{4 \times 15}{5 \times 14}$$

$$= \frac{12 \times 5}{5 \times 18}$$

$$= \frac{\cancel{6} \times 3}{7 \times \cancel{4}}$$

$$= \frac{4 \times \cancel{15}}{\cancel{5} \times 14}$$

$$= \frac{\cancel{12} \times \cancel{5}}{5 \times \cancel{18}}$$

$$= \frac{1 \times 3}{7 \times 1}$$

$$= \frac{1 \times 3}{1 \times 7}$$

$$= \frac{1 \times 1}{1 \times 3}$$

Multiplying Mixed Numbers

Mixed numbers combine whole numbers with fraction parts.

To multiply, write the mixed numbers in fraction form.

Multiply: $2\frac{1}{4} \times \frac{2}{3}$

Rewrite $2\frac{1}{4}$ as an improper fraction: $2\frac{1}{4} = \frac{2 \times 4 + 1}{4} = \frac{9}{4}$

$$\begin{aligned} \text{So, } 2\frac{1}{4} \times \frac{2}{3} &= \frac{9}{4} \times \frac{2}{3} \\ &= \frac{9 \times 2}{4 \times 3} \\ &= \frac{9^3 \times 2^1}{4^2 \times 3^1} \\ &= \frac{3}{2}, \text{ or } 1\frac{1}{2} \end{aligned}$$

Multiply the numerators and multiply the denominators.

Look for common factors in numerator and denominator.

Check

1. Write each mixed number as an improper fraction.

a) $3\frac{4}{5}$

$$\begin{aligned} &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

b) $3\frac{2}{7}$

$$\begin{aligned} &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

c) $1\frac{5}{12}$

$$\begin{aligned} &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

2. Multiply.

a) $3\frac{2}{5} \times \frac{1}{4}$

$$\begin{aligned} &= \frac{17}{5} \times \frac{1}{4} \\ &= \frac{\times}{\times} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

Rewrite $3\frac{2}{5}$ as an improper fraction: $3\frac{2}{5} = \frac{17}{5}$

Multiply the numerators and multiply the denominators.

b) $1\frac{1}{2} \times 1\frac{1}{3}$

$$\begin{aligned} &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}} \end{aligned}$$

Rewrite _____ and _____ as improper fractions.

Multiply the numerators and multiply the denominators.

Look for common factors in numerator and denominator.

3.4 Multiplying Rational Numbers

FOCUS Multiply rational numbers.

To predict the sign of the product of two rational numbers, use the sign rules for multiplying integers:

\times	$(-)$	$(+)$
$(-)$	$(+)$	$(-)$
$(+)$	$(-)$	$(+)$

- If the signs are the same, the answer is positive.
- If the signs are different, the answer is negative.

Example 1 Multiplying Rational Numbers in Fraction Form

Multiply: $\left(-\frac{2}{3}\right)\left(-\frac{6}{7}\right)$

Solution

Predict the sign of the product:

Since the fractions have the same sign, their product is positive.

$$\begin{aligned}\left(-\frac{2}{3}\right)\left(-\frac{6}{7}\right) &= \frac{(-2) \times (-6)}{3 \times 7} \\ &= \frac{(-2) \times (-2)}{1 \times 7} \\ &= \frac{4}{7}\end{aligned}$$

$$\text{So, } \left(-\frac{2}{3}\right)\left(-\frac{6}{7}\right) = \frac{4}{7}$$

Check

1. Find each product.

a) $\frac{1}{5} \times \left(-\frac{3}{5}\right)$

The fractions have _____,
so their product is _____.

$$\begin{aligned}&= \frac{\quad}{\quad} \times \frac{\quad}{\quad} \\ &= \frac{\quad}{\quad}\end{aligned}$$

b) $\left(-\frac{9}{11}\right)\left(-\frac{7}{12}\right)$

$= \frac{\quad}{\quad} \times \frac{\quad}{12}$

$= \frac{\quad}{\quad} \times \frac{\quad}{\quad}$

$= \frac{\quad}{\quad} \times \frac{\quad}{\quad}$

$= \frac{\quad}{\quad}$

The fractions have _____,
so their product is _____.

A common factor of _____ and 12 is _____.

Example 2

Multiplying Rational Numbers in Mixed Number Form

Multiply: $\left(-2\frac{1}{5}\right)\left(-1\frac{3}{4}\right)$

Solution

$\left(-2\frac{1}{5}\right)\left(-1\frac{3}{4}\right)$

Write each mixed number as an improper fraction.

$2\frac{1}{5} = \frac{10}{5} + \frac{1}{5} = \frac{11}{5}$

$1\frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4}$

So, $\left(-2\frac{1}{5}\right)\left(-1\frac{3}{4}\right) = \left(-\frac{11}{5}\right)\left(-\frac{7}{4}\right)$
 $= \frac{(-11) \times (-7)}{5 \times 4}$
 $= \frac{77}{20}$, or $3\frac{17}{20}$

The numbers have the same sign: the product is positive.

$\frac{77}{20} = \frac{60}{20} + \frac{17}{20} = 3\frac{17}{20}$

Check

1. Find each product.

a) $\left(-1\frac{1}{4}\right) \times \frac{6}{7}$

$= \left(-\frac{\quad}{4}\right) \times \frac{6}{7}$

$= \frac{\quad}{\quad} \times \frac{\quad}{\quad}$

$= \frac{\quad}{\quad} \times \frac{\quad}{\quad}$

$= \frac{\quad}{\quad}$, or $\frac{\quad}{\quad}$

b) $\left(-2\frac{4}{5}\right)\left(-2\frac{3}{4}\right)$

$= \left(-\frac{\quad}{5}\right)\left(-\frac{\quad}{4}\right)$

$= \frac{\quad}{\quad} \times \frac{\quad}{\quad}$

$= \frac{\quad}{\quad}$

$= \frac{\quad}{\quad}$

To multiply rational numbers in decimal form:

- Use the sign rules for integers to find the sign of the product.
- Multiply as you would with whole numbers; estimate to place the decimal point.

Example 3 Multiplying Rational Numbers to Solve a Problem

On March 6, 2009, the price of a share in Bank of Montreal changed by $-\$3.05$. Joanne owns 50 shares. By how much did the shares change in value that day?

Solution

The change in value is: $50 \times (-3.05)$

Multiply the integers, then estimate to place the decimal point.

$$50 \times (-305) = -15\,250$$

Estimate to place the decimal point.

Since -3.05 is close to -3 ,

$50 \times (-3.05)$ is close to $50 \times (-3)$, or -150 .

So, $50 \times (-3.05) = -152.50$

The product is negative.

The shares changed in value by $-\$152.50$ that day.

Check

1. On March 13, 2009, the price of a share in Research in Motion changed by $-\$1.13$. Tania owns 80 shares. By how much did those shares change in value that day?

The change in value is: $80 \times (-1.13)$

The product is _____.

To find $80 \times (-1.13)$, multiply: _____ \times _____

$80 \times$ _____ $=$ _____

Estimate: $80 \times (-1.13)$ is about _____ \times _____ $=$ _____

So, $80 \times (-1.13) =$ _____

The shares changed in value by _____ that day.

Practice

1. Is the product positive or negative?

- a) $(-2.5) \times 3.6$ different signs; the product is _____.
- b) $(-4.1) \times (-6.8)$ the same sign; the product is _____.
- c) $\left(-\frac{3}{4}\right)\left(-\frac{7}{9}\right)$ _____; the product is _____.
- d) $\left(-2\frac{1}{3}\right) \times 6\frac{1}{2}$ _____; the product is _____.

2. Which of these expressions have the same product as $\frac{5}{8} \times \left(-\frac{7}{3}\right)$? Why?

- a) $\left(-\frac{7}{3}\right) \times \frac{5}{8}$ _____, since _____

- b) $\left(-\frac{5}{8}\right)\left(-\frac{7}{3}\right)$ _____, since _____

- c) $\frac{7}{3} \times \frac{5}{8}$ _____, since _____

- d) $\frac{7}{3} \times \left(-\frac{5}{8}\right)$ _____, since _____

3. Find each product.

a) $\frac{2}{7} \times \left(-\frac{5}{6}\right)$
 $\frac{2}{7} \times \left(-\frac{5}{6}\right) =$ _____
 = _____
 = _____
 = _____

b) $\left(-\frac{4}{5}\right)\left(-\frac{11}{12}\right)$
 $\left(-\frac{4}{5}\right)\left(-\frac{11}{12}\right) =$ _____
 = _____
 = _____
 = _____

Think: Is the product positive or negative?

4. Find each product.

a) $\left(-\frac{8}{9}\right) \times 1\frac{1}{2}$

$$\left(-\frac{8}{9}\right) \times 1\frac{1}{2} = \left(-\frac{8}{9}\right) \times \frac{\quad}{2}$$

$$= \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

b) $\left(-2\frac{5}{6}\right)\left(-1\frac{1}{5}\right)$

$$\left(-2\frac{5}{6}\right)\left(-1\frac{1}{5}\right) = \left(-\frac{\quad}{6}\right)\left(-\frac{\quad}{5}\right)$$

$$= \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

5. Multiply.

a) $0.4 \times (-3.2)$

To find $0.4 \times (-3.2)$, multiply: $4 \times (-32) = \underline{\hspace{2cm}}$

$0.4 \times (-3.2)$ is about $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

So, $0.4 \times (-3.2) = \underline{\hspace{2cm}}$.

b) $(-3.03) \times (-0.7)$

To find $(-3.03) \times (-0.7)$, multiply: $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

$(-3.03) \times (-0.7)$ is about $(\underline{\hspace{1cm}}) \times (\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$

So, $(-3.03) \times (-0.7) = \underline{\hspace{2cm}}$.

6. On a certain day, the temperature changed by an average of $-2.2^\circ\text{C}/\text{h}$.
What was the total temperature change in 8 h?

The total change in temperature is: $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

The product is $\underline{\hspace{2cm}}$.

To find $\underline{\hspace{1cm}}$, multiply: $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

$8 \times (-2.2)$ is about $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$.

So, $8 \times (-2.2) = \underline{\hspace{2cm}}$

The temperature $\underline{\hspace{1cm}}$ by $\underline{\hspace{1cm}}^\circ\text{C}$ in 8 h.