

2.5 Skill Builder

Grouping Equal Factors

In multiplication, you can group equal factors.

For example:

$$\begin{aligned} & 3 \times 7 \times 7 \times 3 \times 7 \times 7 \times 3 \\ &= \underbrace{3 \times 3 \times 3}_{3^3} \times \underbrace{7 \times 7 \times 7 \times 7}_{7^4} \\ &= 3^3 \times 7^4 \end{aligned}$$

Group equal factors.

Write repeated multiplication as powers.

*Order does not
matter in
multiplication.*

Check

1. Group equal factors and write as powers.

a) $2 \times 10 \times 2 \times 10 \times 2 = \underline{2 \times 2 \times 2 \times 10 \times 10}$
 $= \underline{\hspace{2cm}}$

b) $2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 = \underline{\hspace{2cm}}$
 $= \underline{\hspace{2cm}}$

Multiplying Fractions

To multiply fractions, first multiply the numerators, and then multiply the denominators.

$$\begin{aligned} \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} &= \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} \\ &= \frac{2^4}{3^4} \end{aligned}$$

Write repeated multiplication as powers.

*There are 4 factors of 2, and
4 factors of 3.*

Check

1. Multiply the fractions. Write as powers.

a) $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \underline{\hspace{2cm}}$
 $= \underline{\hspace{2cm}}$

b) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \underline{\hspace{2cm}}$
 $= \underline{\hspace{2cm}}$
 $= \underline{\hspace{2cm}}$

2.5 Exponent Laws II

FOCUS Understand and apply exponent laws for powers of: products; quotients; and powers.

$$\begin{aligned} &\text{Multiply } 3^2 \times 3^2 \times 3^2. \\ &3^2 \times 3^2 \times 3^2 = 3^{2+2+2} \\ &= 3^6 \end{aligned}$$

We can write repeated multiplication as powers.

$$\begin{aligned} &\text{So, } \underbrace{3^2 \times 3^2 \times 3^2}_{3 \text{ factors of } (3^2)} \end{aligned}$$

$$\begin{aligned} &= (3^2)^3 \\ &= 3^6 \end{aligned}$$

$$2 \times 3 = 6$$

$$\begin{aligned} &\text{We write: } (3^2)^3 = 3^{2 \times 3} \\ &= 3^6 \end{aligned}$$

Use the exponent law for the product of powers.
Add the exponents.

The base is 3^2 .

The exponent is 3.

This is a **power of a power**.

Look at the pattern in the exponents.

This is also a power.

Exponent Law for a Power of a Power

To raise a power to a power, multiply the exponents.

For example: $(2^3)^5 = 2^{3 \times 5}$

Example 1 Simplifying a Power of a Power

Write as a power.

a) $(3^2)^4$

b) $[(-5)^3]^2$

c) $-(2^3)^4$

Solution

Use the exponent law for a power of a power: multiply the exponents.

$$\begin{aligned} \text{a) } (3^2)^4 &= 3^{2 \times 4} \\ &= 3^8 \end{aligned}$$

$$\begin{aligned} \text{b) } [(-5)^3]^2 &= (-5)^{3 \times 2} && \text{The base is } -5. \\ &= (-5)^6 \end{aligned}$$

$$\begin{aligned} \text{c) } -(2^3)^4 &= -(2^{3 \times 4}) && \text{The base is } 2. \\ &= -2^{12} \end{aligned}$$

Check

1. Write as a power.

a) $(9^3)^4 = 9 \times \dots$
 $= 9 \dots$

b) $[(-2)^5]^3 = (-2) \dots$
 $= (-2) \dots$

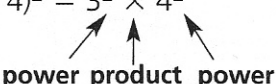
c) $-(5^4)^2 = -(5 \dots)$
 $= -5 \dots$

Multiply $(3 \times 4)^2$.

Write as repeated multiplication.

$$\begin{aligned}(3 \times 4)^2 &= (3 \times 4) \times (3 \times 4) \\ &= 3 \times 4 \times 3 \times 4 \\ &= (\underbrace{3 \times 3}) \times (\underbrace{4 \times 4}) \\ &\quad \text{2 factors of 3} \quad \text{2 factors of 4} \\ &= 3^2 \times 4^2\end{aligned}$$

So, $(3 \times 4)^2 = 3^2 \times 4^2$



The base of the power is a product: $\underbrace{3 \times 4}_{\text{base}}$

Remove the brackets.

Group equal factors.

Write as powers.

Exponent Law for a Power of a Product

The power of a product is the product of powers.

For example: $(2 \times 3)^4 = 2^4 \times 3^4$

Example 2 Evaluating Powers of Products

Evaluate.

a) $(2 \times 5)^2$

b) $[(-3) \times 4]^2$

Solution

Use the exponent law for a power of a product.

$$\begin{aligned}\text{a) } (2 \times 5)^2 &= 2^2 \times 5^2 \\ &= (2)(2) \times (5)(5) \\ &= 4 \times 25 \\ &= 100\end{aligned}$$

$$\begin{aligned}\text{b) } [(-3) \times 4]^2 &= (-3)^2 \times 4^2 \\ &= (-3)(-3) \times (4)(4) \\ &= 9 \times 16 \\ &= 144\end{aligned}$$

Or, use the order of operations and evaluate what is inside the brackets first.

$$\begin{aligned}\text{a) } (2 \times 5)^2 &= 10^2 \\ &= 100\end{aligned}$$

$$\begin{aligned}\text{b) } [(-3) \times 4]^2 &= (-12)^2 \\ &= 144\end{aligned}$$

Check

1. Write as a product of powers.

a) $(5 \times 7)^4 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

b) $(8 \times 2)^2 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$

2. Evaluate.

a) $[(-1) \times 6]^2 = \underline{\hspace{1cm}}^2$
 $= \underline{\hspace{1cm}}$

b) $[(-1) \times (-4)]^3 = \underline{\hspace{1cm}}^3$
 $= \underline{\hspace{1cm}}$

Evaluate $\left(\frac{3}{4}\right)^2$.
base

The base of the power is a quotient: $\frac{3}{4}$

Write as repeated multiplication.

$$\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right) \times \left(\frac{3}{4}\right)$$

$$= \frac{3}{4} \times \frac{3}{4}$$

$$= \frac{3 \times 3}{4 \times 4}$$

$$= \frac{3^2}{4^2}$$

Multiply the fractions.

Write repeated multiplication as powers.

So, $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2}$

power
quotient
power

Exponent Law for a Power of a Quotient

The power of a quotient is the quotient of powers.

For example: $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$

Example 3 Evaluating Powers of Quotients

Evaluate.

a) $[30 \div (-5)]^2$

b) $\left(\frac{20}{4}\right)^2$

Solution

Use the exponent law for a power of a quotient.

$$\begin{aligned}\text{a) } [30 \div (-5)]^2 &= \left(\frac{30}{-5}\right)^2 \\ &= \frac{30^2}{(-5)^2} \\ &= \frac{900}{25} \\ &= 36\end{aligned}$$

$$\begin{aligned}\text{b) } \left(\frac{20}{4}\right)^2 &= \frac{20^2}{4^2} \\ &= \frac{400}{16} \\ &= 25\end{aligned}$$

Or, use the order of operations and evaluate what is inside the brackets first.

$$\begin{aligned}\text{a) } [30 \div (-5)]^2 &= (-6)^2 \\ &= 36\end{aligned}$$

$$\begin{aligned}\text{b) } \left(\frac{20}{4}\right)^2 &= 5^2 \\ &= 25\end{aligned}$$

Check

1. Write as a quotient of powers.

$$\text{a) } \left(\frac{3}{4}\right)^5 = \underline{\hspace{2cm}}$$

$$\text{b) } [1 \div (-10)]^3 = \underline{\hspace{2cm}}$$

2. Evaluate.

$$\begin{aligned}\text{a) } [(-16) \div (-4)]^2 \\ = \underline{\hspace{1cm}}^2 = \underline{\hspace{1cm}}\end{aligned}$$

$$\begin{aligned}\text{b) } \left(\frac{36}{6}\right)^3 &= \underline{\hspace{2cm}} \\ &= \underline{\hspace{2cm}}\end{aligned}$$

You can
evaluate what
is inside the
brackets first.

Practice

1. Write as a product of powers.

$$\text{a) } (5 \times 2)^4 = 5 \underline{\hspace{1cm}} \times 2 \underline{\hspace{1cm}}$$

$$\text{b) } (12 \times 13)^2 = \underline{\hspace{2cm}}$$

$$\text{c) } [3 \times (-2)]^3 = \underline{\hspace{2cm}}$$

$$\text{d) } [(-4) \times (-5)]^5 = \underline{\hspace{2cm}}$$

2. Write as a quotient of powers.

$$\text{a) } (5 \div 8)^0 = \underline{\hspace{2cm}}$$

$$\text{b) } [(-6) \div 5]^7 = \underline{\hspace{2cm}}$$

$$\text{c) } \left(\frac{3}{5}\right)^2 = \underline{\hspace{2cm}}$$

$$\text{d) } \left(\frac{-1}{-2}\right)^3 = \underline{\hspace{2cm}}$$

3. Write as a power.

a) $(5^2)^3 = 5 \times \underline{\hspace{1cm}}$
 $= 5 \underline{\hspace{1cm}}$

b) $[(-2)^3]^5 = (-2) \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

c) $(4^4)^1 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

d) $(8^0)^3 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

4. Evaluate.

a) $[(6 \times (-2))^2] = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

b) $-(3 \times 4)^2 = -(\underline{\hspace{1cm}}) \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

c) $\left(\frac{-8}{-2}\right)^2 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

d) $(10 \times 3)^1 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

e) $[(-2)^1]^2 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

f) $[(-2)^1]^3 = \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$

5. Find any errors and correct them.

a) $(3^2)^3 = 3^5$ $\underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}}$

b) $(3 + 2)^2 = 3^2 + 2^2$ $\underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}}$

c) $(5^3)^3 = 5^9$ $\underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}}$

d) $\left(\frac{2}{3}\right)^8 = \frac{2^8}{3^8}$ $\underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}}$

e) $(3 \times 2)^2 = 36$ $\underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}}$

f) $\left(\frac{2}{3}\right)^2 = \frac{4}{6}$ $\underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}}$

g) $[(-3)^3]^0 = (-3)^3$ $\underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}}$
 $\underline{\hspace{1cm}}$

h) $[(-2) \times (-3)]^4 = -6^4$ $\underline{\hspace{1cm}}$