Powers and Exponent Laws

What You'll Learn

- Use powers to show repeated multiplication.
- Evaluate powers with exponent 0.
- Write numbers using powers of 10.
- Use the order of operations with exponents.
- Use the exponent laws to simplify and evaluate expressions.

Why It's Important

Powers are used by

- lab technicians, when they interpret a patient's test results
- reporters, when they write large numbers in a news story

Key Words

integer exponent

opposite squared positive cubed

negative standard form

factor product power quotient

base

Multiplying Integers

When multiplying 2 integers, look at the sign of each integer:

- When the integers have the same sign, their product is positive.
- When the integers have different signs, their product is negative.

6	\times	(-	-3)

These 2 integers have different signs, so their product is negative.

$$6 \times (-3) = -18$$

$$(-10) \times (-2)$$

 $(-10) \times (-2)$ These 2 integers have the same sign, so their product is positive.

$$(-10) \times (-2) = 20$$

(-)(+)× (-)(+)(-)(-)(+)(+)

> When an integer is positive, we do not have to write the + sign in front.

Check

1. Will the product be positive or negative?

a) 7 × 4

- **b)** 3 × (-6)
- **c)** $(-9) \times 10$
- **d)** $(-5) \times (-9)$

2. Multiply.

a) $7 \times 4 =$ _____

b) 3 × (-6) = _____

c) $(-9) \times 10 =$

d) $(-5) \times (-9) =$

e) $(-3) \times (-5) =$

f) $2 \times (-5) =$

g) $(-8) \times 2 =$

h) $(-4) \times 3 =$

Multiplying More than 2 Integers

We can multiply more than 2 integers. Multiply pairs of integers, from left to right.

$$(-1) \times (-2) \times (-3)$$

$$= 2 \times (-3)$$

$$= -6$$

$$(-1) \times (-2) \times (-3) \times (-4)$$

= $2 \times (-3) \times (-4)$
= $(-6) \times (-4)$
= 24

The product of 3 negative factors is negative.

The product of 4 negative factors is positive.

Multiplying Integers

When the number of negative factors is *even*, the product is positive. When the number of negative factors is *odd*, the product is negative.

We can show products of integers in different ways: $(-2) \times (-2) \times 3 \times (-2)$ is the same as (-2)(-2)(3)(-2).

So,
$$(-2) \times (-2) \times 3 \times (-2) = (-2)(-2)(3)(-2)$$

= -24

Check

1. Multiply.

a)
$$(-3) \times (-2) \times (-1) \times 1$$

Is the answer positive or negative? How can you tell?

2.1 What is a Power?

FOCUS Show repeated multiplication as a power.

We can use powers to show repeated multiplication.

$$2 \times 2 \times 2 \times 2 \times 2 = 2^{5}$$
Repeated
Repeated
Power

Replacement.

Power

Repeated

Repeated

Power

Repeated

Power

Power

Repeated

Power

We read 2^5 as "2 to the 5th." Here are some other powers of 2.

5 factors of 2

Repeated Multiplication	Power	Read as
1 factor of 2	21	2 to the 1st
2×2 2 factors of 2	2 ²	2 to the 2nd, or 2 squared
$2 \times 2 \times 2$ 3 factors of 2	23	2 to the 3rd, or 2 cubed
$2 \times 2 \times 2 \times 2$ 4 factors of 2	2 ⁴	2 to the 4th

In each case, the exponent in the power is equal to the number of factors in the repeated multiplication.

Example 1

Writing Powers

Write as a power.

a)
$$4 \times 4 \times 4 \times 4 \times 4 \times 4$$

Solution

$$\underbrace{4 \times 4 \times 4 \times 4 \times 4 \times 4}_{6 \text{ factors of } 4} = 4^{6}$$

So,
$$4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$$

So,
$$3 = 3^1$$

Check

1. Write as a power.

a)
$$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2$$

b)
$$5 \times 5 \times 5 \times 5 = 5$$
—

c)
$$(-10)(-10)(-10) =$$

d)
$$4 \times 4 = 1$$

e)
$$(-7)(-7)(-7)(-7)(-7)(-7)(-7)(-7) =$$

2. Complete the table.

	Repeated Multiplication	Power	Read as
a)	8 × 8 × 8 × 8		8 to the 4th
b)	7 × 7		
c)	3 × 3 × 3 × 3 × 3		3 to the 6th
d)	$2 \times 2 \times 2$		

Power	Repeated Multiplication	Standard Form
2 ⁵	$2 \times 2 \times 2 \times 2 \times 2$	32

Example 2 Evaluating Powers

Write as repeated multiplication and in standard form.

Solution

a)
$$2^4 = 2 \times 2 \times 2 \times 2$$
 As repeated multiplication
= 16 Standard form

b)
$$5^3 = 5 \times 5 \times 5$$
 As repeated multiplication = 125 Standard form

1. Complete the table.

Power	Repeated Multiplication	Standard Form
2 ³	2 × 2 × 2	
6 ²		36
3 ⁴	*	
104		
8 squared		
7 cubed		

To evaluate a power that contains negative integers, identify the base of the power. Then, apply the rules for multiplying integers.

Example 3

Evaluating Expressions Involving Negative Signs

Identify the base, then evaluate each power.

a)
$$(-5)^4$$

b)
$$-5^4$$

Solution

a)
$$(-5)^4$$

The brackets tell us that the base of this power is (-5).

$$(-5)^4 = (-5) \times (-5) \times (-5) \times (-5)$$

There is an even number of negative integers, so the product is positive.

b)
$$-5^4$$

There are no brackets. So, the base of this power is 5. The negative sign applies to the whole expression.

$$-5^4 = -(5 \times 5 \times 5 \times 5)$$

= -625

- **1.** Identify the base of each power, then evaluate.
 - **a)** $(-1)^3$

The base is _____.

 $(-1)^3 =$ _____

- c) $(-7)^2$ The base is _____. $(-7)^2 =$ ______
- **b)** -10^3

The base is _____.

 $-10^3 =$ _____

d) −(−5)⁴

The base is _____. $-(-5)^4 =$ _____.

The first negative sign applies to the whole expression.

Practice

- 1. Write as a power.

The base is 8. There are _____ equal factors, so the exponent is _____

 $8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8$ —

b) $10 \times 10 \times 10 \times 10 \times 10$

5 factors of 10

The base is _____. There are _____ equal factors, so the exponent is _____.

So, $10 \times 10 \times 10 \times 10 \times 10 =$

c) (-2)(-2)(-2)3 factors of ____

The base is _____. There are _____ equal factors, so the exponent is _____.

So, (-2)(-2)(-2) =

d) (-13)(-13)(-13)(-13)(-13)

_____factors of _____

The base is _____. There are _____ equal factors, so the exponent is _____.

So, (-13)(-13)(-13)(-13)(-13)(-13) =

- 2. Write each expression as a power.
 - **a)** $9 \times 9 \times 9 \times 9 = ___4$
- **b)** (5)(5)(5)(5)(5) = 5

c) 11 × 11 = ____

d) (-12)(-12)(-12)(-12) =

NACE and noway as reported	multiplication	
3. Write each power as repeated r		
a) $3^2 = $	b) 3 ⁴ =	% tolerately the mass <u>cf.c.</u>
c) $2^7 = $		
d) $10^8 = $	a standard .	
4. State whether the answer will b	pe positive or negative.	Identify the base first.
a) (-3) ²	b) 6 ³	
c) (-10) ³	d) -4 ³	
5. Write each power as repeated	multiplication and in standard for	m.
a) $(-3)^2 = $	b) 6 ³ =	Predict. Will the answer be positive of negative?
c) $(-10)^3 =$	d) $-4^3 =$	
6. Write each product as a power	and in standard form.	
a) $(-3)(-3)(-3) = $	b) $(-8)(-8) = $	
	· · · · · · · · · · · · · · · · · · ·	
c) $-(8 \times 8 \times 8) = $	d) $-(-1)(-1)(-1)(-1)(-1)(-1)(-1)(-1)(-1)(-1$	1)(-1)(-1)(-1) =
=		10 786766 1
7. Identify any errors and correct	them.	
a) $4^3 = 12$		
b) $(-2)^9$ is negative		(Sr Az As 1) (4)
suasoopa adi	There are — ercei factors, so	al ens. Terr
c) $(-9)^2$ is negative		20,1-23(-23) -23
	(-13)(-13)(-13)	
d) $3^2 = 2^3$		vactors of

e) $(-10)^2 = 100$