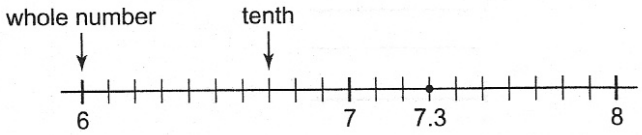
1.2 Skill Builder

Degree of Accuracy

We are often asked to write an answer to a given decimal place.
To do this, we can use a number line.

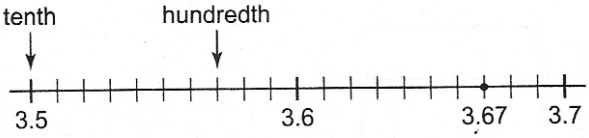
To write 7.3 to the nearest whole number:
Place 7.3 on a number line in tenths.



3 is the last digit. It is in the tenths position. So, use a number line in tenths.

7.3 is closer to 7 than to 8.
So, 7.3 to the nearest whole number is: 7

To write 3.67 to the nearest tenth:
Place 3.67 on a number line in hundredths.



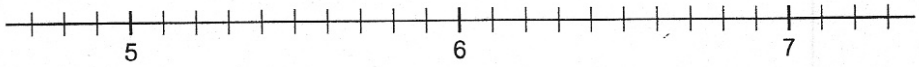
7 is the last digit. It is in the hundredths position. So, use a number line in hundredths.

3.67 is closer to 3.7 than to 3.6.
So, 3.67 to the nearest tenth is: 3.7

Check

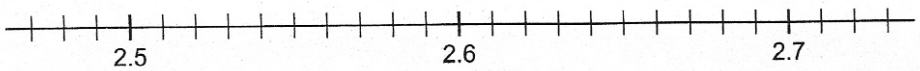
1. Write each number to the nearest whole number.
Mark it on the number line.

a) 5.3 ____ **b)** 6.8 ____ **c)** 7.1 ____ **d)** 6.4 ____



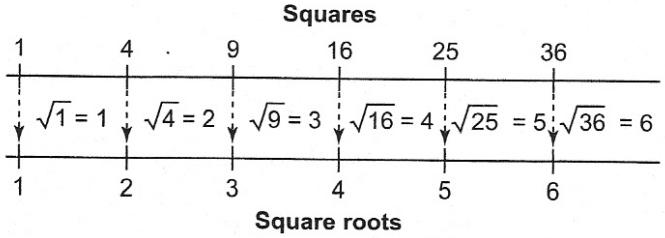
2. Write each number to the nearest tenth.
Mark it on the number line.

a) 2.53 ____ **b)** 2.64 ____ **c)** 2.58 ____ **d)** 2.66 ____

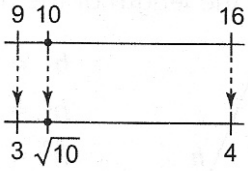


Squares and Square Roots on Number Lines

Most numbers are not perfect squares.
You can use number lines to estimate the square roots of these numbers.



10 is between the perfect squares 9 and 16.
So, $\sqrt{10}$ is between $\sqrt{9}$ and $\sqrt{16}$.
$\sqrt{9} = 3$ and $\sqrt{16} = 4$
So, $\sqrt{10}$ is between 3 and 4.



10 is closer to 9 than 16, so $\sqrt{10}$ is closer to 3 than 4.

Check with a calculator.
$\sqrt{10} \doteq 3.2$, which is between 3 and 4.

Check

1. Between which 2 consecutive whole numbers is each square root?
Explain.

Refer to the squares and square roots number lines.

a) $\sqrt{22}$

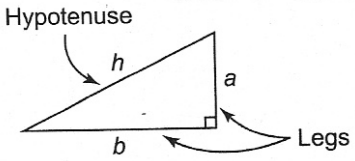
22 is between the perfect squares 16 and 25.
So, $\sqrt{22}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
$\sqrt{\_\_\_\_} = \_\_\_\_$ and $\sqrt{\_\_\_\_} = \_\_\_\_$
So, $\sqrt{22}$ is between _____ and _____.

b) $\sqrt{6}$

6 is between the perfect squares _____ and _____.
So, $\sqrt{6}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
$\sqrt{\_\_\_\_} = \_\_\_\_$ and $\sqrt{\_\_\_\_} = \_\_\_\_$
So, $\sqrt{6}$ is between _____ and _____.

The Pythagorean Theorem

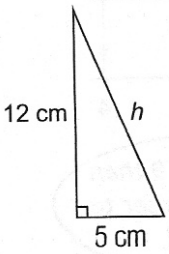
You can use the Pythagorean Theorem to find unknown lengths in right triangles.



Pythagorean Theorem

$h^2 = a^2 + b^2$

To find the length of the hypotenuse, h, in this triangle:



$h^2 = 5^2 + 12^2$

$h^2 = 25 + 144$

$h^2 = 169$

$h = \sqrt{169}$

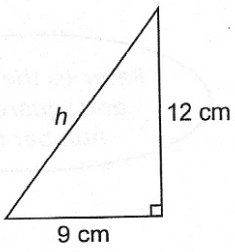
$h = 13$

The length of the hypotenuse is 13 cm.

Check

1. Use the Pythagorean Theorem to find the length of each hypotenuse, h.

a)



$h^2 =$ ____ + ____

$h^2 =$ ____ + ____

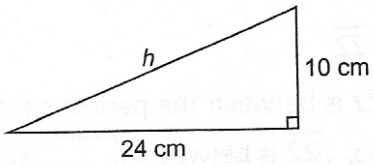
$h^2 =$ ____

$h = \sqrt{}$

$h =$ ____

The length of the hypotenuse is ____ cm.

b)



$h^2 =$ ____ + ____

$h^2 =$ ____ + ____

$h^2 =$ ____

$h = \sqrt{}$

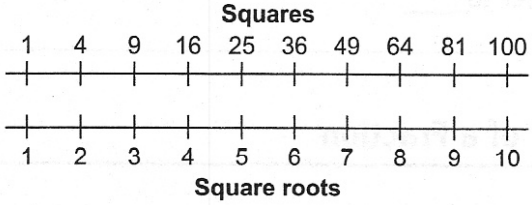
$h =$ ____

The length of the hypotenuse is ____ cm.

1.2 Square Roots of Non-Perfect Squares

FOCUS **Approximate the square roots of decimals and fractions that are not perfect squares.**

The top number line shows all the perfect squares from 1 to 100.



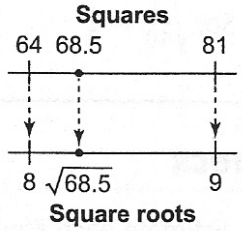
The bottom number line shows the square root of each number in the top line. You can use these lines to estimate the square roots of fractions and decimals that are not perfect squares.

Example 1 Estimating a Square Root of a Decimal

Estimate: $\sqrt{68.5}$

Solution

68.5 is between the perfect squares 64 and 81.
So, $\sqrt{68.5}$ is between $\sqrt{64}$ and $\sqrt{81}$.
That is, $\sqrt{68.5}$ is between 8 and 9.
Since 68.5 is closer to 64 than 81, $\sqrt{68.5}$ is closer to 8 than 9.
So, $\sqrt{68.5}$ is between 8 and 9, and closer to 8.



Check

1. Estimate each square root.
Explain your estimate.

a) $\sqrt{13.5}$

13.5 is between the perfect squares ____ and ____.
So, $\sqrt{13.5}$ is between $\sqrt{}$ and $\sqrt{}$.
That is, $\sqrt{13.5}$ is between ____ and ____.
Since 13.5 is closer to ____ than ____, $\sqrt{13.5}$ is closer to ____ than ____.
So, $\sqrt{13.5}$ is between ____ and ____, and closer to ____.

b) $\sqrt{51.5}$

51.5 is between the perfect squares _____ and _____.

So, $\sqrt{51.5}$ is between $\sqrt{\quad}$ and $\sqrt{\quad}$.

That is, $\sqrt{51.5}$ is between ____ and ____.

Since 51.5 is closer to ____ than ____, $\sqrt{51.5}$ is closer to ____ than ____.

So, $\sqrt{51.5}$ is between ____ and ____, and closer to ____.

Example 2 Estimating a Square Root of a Fraction

Estimate: $\sqrt{\frac{3}{10}}$

Solution

Find the closest perfect square to the numerator and denominator.

In the fraction $\frac{3}{10}$:

3 is close to the perfect square 4.

10 is close to the perfect square 9.

So, $\sqrt{\frac{3}{10}} \doteq \sqrt{\frac{4}{9}}$ and $\sqrt{\frac{4}{9}} = \frac{2}{3}$

So, $\sqrt{\frac{3}{10}} \doteq \frac{2}{3}$

Check

1. Estimate each square root.

a) $\sqrt{\frac{23}{80}}$

23 is close to the perfect square ____.

80 is close to the perfect square ____.

So, $\sqrt{\frac{23}{80}} \doteq \sqrt{\frac{\quad}{\quad}}$

$\sqrt{\frac{\quad}{\quad}} = \frac{\quad}{\quad}$

So, $\sqrt{\frac{23}{80}} \doteq$ ____

b) $\sqrt{\frac{8}{17}}$

8 is close to the perfect square ____.

17 is close to the perfect square ____.

So, $\sqrt{\frac{8}{17}} \doteq \sqrt{\frac{\quad}{\quad}}$

$\sqrt{\frac{\quad}{\quad}} = \frac{\quad}{\quad}$

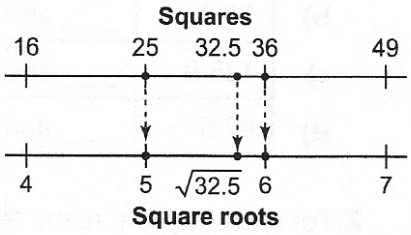
So, $\sqrt{\frac{8}{17}} \doteq$ _____

Example 3 Finding a Number with a Square Root between Two Given Numbers

Identify a decimal that has a square root between 5 and 6.

Solution

$5^2 = 25$, so 5 is a square root of 25.
$6^2 = 36$, so 6 is a square root of 36.
So, any decimal between 25 and 36 has a square root between 5 and 6.
Choose 32.5.



Check the answer by using a calculator.
$\sqrt{32.5} \doteq 5.7$, which is between 5 and 6.
So, the decimal 32.5 is one correct answer.
There are many more correct answers.

Check

1. a) Identify a decimal that has a square root between 7 and 8.
Check the answer.
$7^2 =$ _____ and $8^2 =$ _____
So, any decimal between _____ and _____ has a square root between 7 and 8.
Choose _____.
Check the answer on a calculator.
$\sqrt{} \doteq$ _____
The decimal _____ is one correct answer.

b) Identify a decimal that has a square root between 11 and 12.
_____ = _____ and _____ = _____
So, any decimal between _____ and _____ has a square root between 11 and 12.
Choose _____.
$\sqrt{} \doteq$ _____
So, _____ is one correct answer.

Practice

1. For each number, name the 2 closest perfect squares and their square roots.

	Number	Two closest perfect squares	Their square roots
a)	44.4	____ and ____	____ and ____
b)	10.8	____ and ____	____ and ____
c)	125.9	____ and ____	____ and ____
d)	87.5	____ and ____	____ and ____

2. For each fraction, name the closest perfect square and its square root for the numerator and for the denominator.

	Fraction	Closest perfect squares	Their square roots
a)	$\frac{5}{11}$	Numerator: ____; denominator: ____	____ and ____
b)	$\frac{17}{45}$	Numerator: ____; denominator: ____	____ and ____
c)	$\frac{3}{24}$	Numerator: ____; denominator: ____	____ and ____
d)	$\frac{11}{62}$	Numerator: ____; denominator: ____	____ and ____

3. Estimate each square root.
Explain.

a) $\sqrt{1.6}$

1.6 is between ____ and ____.
So, $\sqrt{1.6}$ is between $\sqrt{}$ and $\sqrt{}$.
That is, $\sqrt{1.6}$ is between ____ and ____.
Since 1.6 is closer to ____ than ____, $\sqrt{1.6}$ is closer to ____ than ____.
So, $\sqrt{1.6}$ is between ____ and ____, and closer to ____.

b) $\sqrt{44.5}$

44.5 is between ____ and ____.
So, $\sqrt{44.5}$ is between $\sqrt{}$ and $\sqrt{}$.
That is, $\sqrt{44.5}$ is between ____ and ____.
Since 44.5 is closer to ____ than ____, $\sqrt{44.5}$ is closer to ____ than ____.
So, $\sqrt{44.5}$ is between ____ and ____, and closer to ____.

c) $\sqrt{75.8}$

75.8 is between ____ and ____.

So, $\sqrt{75.8}$ is between $\sqrt{}$ and $\sqrt{}$.

That is, $\sqrt{75.8}$ is between ____ and ____.

Since 75.8 is closer to ____ than ____, $\sqrt{75.8}$ is closer to ____ than ____.

So, $\sqrt{75.8}$ is between ____ and ____, and closer to ____.

4. Estimate each square root. Explain.

a) $\sqrt{\frac{7}{15}}$

7 is close to ____; 15 is close to ____.

So, $\sqrt{\frac{7}{15}} \doteq \sqrt{\frac{}{}}$

$\doteq$ ____

b) $\sqrt{\frac{2}{7}}$

2 is close to ____; 7 is close to ____.

So, $\sqrt{\frac{2}{7}} \doteq \sqrt{\frac{}{}}$

$\doteq$ ____

c) $\sqrt{\frac{35}{37}}$

35 is close to ____; 37 is close to ____.

So, $\sqrt{\frac{35}{37}} \doteq \sqrt{\frac{}{}}$

$\doteq$ ____

d) $\sqrt{\frac{99}{122}}$

99 is close to ____; 122 is close to ____.

So, $\sqrt{\frac{99}{122}} \doteq \sqrt{\frac{}{}}$

$\doteq$ ____

5. Identify a decimal that has a square root between the two given numbers. Check the answer.

a) 1 and 2

$1^2 =$ _____ and $2^2 =$ _____

So, any number between _____ and _____ has a square root between 1 and 2.

Choose _____.

Check: $\sqrt{} \doteq$ _____

The decimal _____ is one possible answer.

b) 8 and 9

$8^2 =$ _____ and $9^2 =$ _____

So, any number between _____ and _____ has a square root between 8 and 9.

Choose _____.

Check: $\sqrt{} \doteq$ _____

The decimal _____ is one possible answer.

c) 2.5 and 3.5

_____ = _____ and _____ = _____

So, any number between _____ and _____ has a square root between 2.5 and 3.5.

Choose _____.

Check: $\sqrt{}$ _____ ≐ _____

The decimal _____ is one correct answer.

d) 20 and 21

_____ = _____ and _____ = _____

So, any number between _____ and _____ has a square root between 20 and 21.

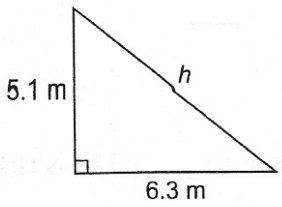
Choose _____.

Check: $\sqrt{}$ _____ ≐ _____

The decimal _____ is one correct answer.

6. Determine the length of the hypotenuse in each right triangle.
Write each answer to the nearest tenth.

a)



$h^2 = 5.1^2 + 6.3^2$

h^2 = _____ + _____

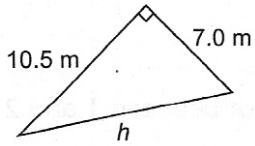
h^2 = _____

$h = \sqrt{}$ _____

$h \doteq$ _____

So, h is about _____ m.

b)



h^2 = _____ + _____

h^2 = _____ + _____

h^2 = _____

$h = \sqrt{}$ _____

$h \doteq$ _____

So, h is about _____ m.

CHECKPOINT

Can you ...

- Identify decimals and fractions that are perfect squares?
- Find the square roots of decimals and fractions that are perfect squares?
- Approximate the square roots of decimals and fractions that are not perfect squares?

1.1 **1.** Calculate the number whose square root is:

a) $\frac{2}{7}$

$\frac{2}{7} \times \frac{2}{7} =$ ____

$\frac{2}{7}$ is a square root of ____.

b) $\frac{8}{11}$

____ × ____ = ____

$\frac{8}{11}$ is a square root of ____.

c) 0.1

____ × ____ = ____

0.1 is a square root of ____.

d) 1.4

1.4 × 1.4 = ____

1.4 is a square root of ____.

2. Identify the fractions that are perfect squares.
The first one has been done for you.

	Fraction	Is numerator a perfect square?	Is denominator a perfect square?	Is fraction a perfect square?
a)	$\frac{64}{75}$	Yes; 8 × 8 = 64	No	No
b)	$\frac{9}{25}$			
c)	$\frac{25}{55}$			

3. Find each square root.

a) $\sqrt{\frac{9}{49}} = \sqrt{\frac{\times}{\_\_ \times \_\_}}$

$=$ ____

b) $\sqrt{\frac{16}{25}} = \sqrt{\frac{\times}{\_\_ \times \_\_}}$

$=$ ____

c) $\sqrt{\frac{36}{121}} = \sqrt{\frac{\times}{\_\_ \times \_\_}}$

$=$ ____

4. a) Put a check mark beside each decimal that is a perfect square.

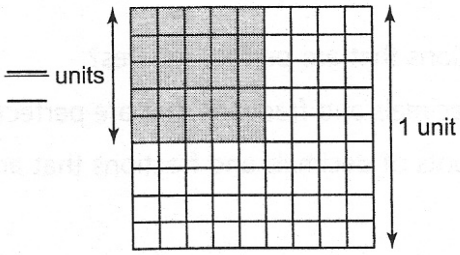
i) 4.84 ____ **ii)** 3.63 ____ **iii)** 98.01 ____ **iv)** 67.24 ____

b) Explain how you identified the perfect squares in part a.

_____

_____

5. a) Find the area of the shaded square.



Area = (Length)^2

$= (\quad)^2$

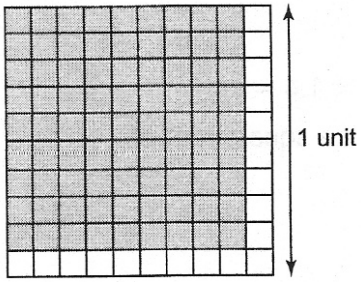
$= \quad \times$

$= \quad$

The area is ____ square units.

b) Find the side length of the shaded square.

Area = $\frac{81}{100}$ square units



Length = $\sqrt{\text{Area}}$

$= \sqrt{\quad}$

$= \sqrt{\quad \times \quad}$

$= \quad$

The side length is ____ units.

1.2 **6.** Estimate each square root.
Explain.

a) $\sqrt{7.5}$

7.5 is between _____ and _____.
So, $\sqrt{7.5}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
That is, $\sqrt{7.5}$ is between _____ and _____.
Since 7.5 is closer to _____ than _____, $\sqrt{7.5}$ is closer to _____ than _____.
So, $\sqrt{7.5}$ is between _____ and _____, and closer to _____.

b) $\sqrt{66.6}$

66.6 is between _____ and _____.
So, $\sqrt{66.6}$ is between $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$.
That is, $\sqrt{66.6}$ is between _____ and _____.
Since 66.6 is closer to _____ than _____, $\sqrt{66.6}$ is closer to _____ than _____.
So, $\sqrt{66.6}$ is between _____ and _____, and closer to _____.

7. Estimate each square root.

a) $\sqrt{\frac{15}{79}}$

15 is close to ____; 79 is close to ____.

So, $\sqrt{\frac{15}{79}} \doteq \sqrt{\frac{\quad}{\quad}}$

$\doteq \frac{\quad}{\quad}$

b) $\sqrt{\frac{23}{50}}$

23 is close to ____; 50 is close to ____.

So, $\sqrt{\frac{23}{50}} \doteq \sqrt{\frac{\quad}{\quad}}$

$\doteq \frac{\quad}{\quad}$

8. Identify a decimal whose square root is between the given numbers. Check your answer.

a) 2 and 3

$2^2 =$ _____ and $3^2 =$ _____

So, any number between _____ and _____ has a square root between 2 and 3.

Choose _____.

Check: $\sqrt{\quad} \doteq$ _____

The decimal _____ is one correct answer.

b) 6 and 7

$6^2 =$ _____ and $7^2 =$ _____

So, any number between _____ and _____ has a square root between 6 and 7.

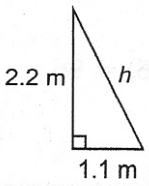
Choose _____.

$\sqrt{\quad} \doteq$ _____

The decimal _____ is one correct answer.

9. Find the length of each hypotenuse.

a)



$h^2 =$ _____ + _____

$h^2 =$ _____ + _____

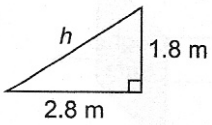
$h^2 =$ _____

$h = \sqrt{\quad}$

$h \doteq$ _____

The length of the hypotenuse is about _____ m.

b)



$h^2 =$ _____ + _____

$h^2 =$ _____ + _____

$h^2 =$ _____

$h = \sqrt{\quad}$

$h \doteq$ _____

The length of the hypotenuse is about _____ m.