UNIT 1

Square Roots and Surface Area

What You'll Learn

- Find square roots of fractions and decimals that are perfect squares.
- Approximate the square roots of fractions and decimals that are not perfect squares.
- Find the surface areas of composite objects.

Why It's Important

Square roots are used by

- police officers, to estimate the speed of a vehicle when it crashed
- vets, to calculate drug dosages

Surface area is used by

- painters, to find the number of cans of paint needed to paint a room
- farmers, to find the amount of fertilizer needed for a field

Key Words

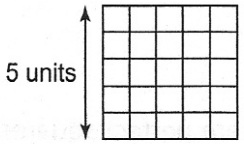
square
square root
perfect square
non-perfect square
terminating decimal
repeating decimal
non-terminating, non-repeating decimal
surface area
composite object

1.1 Skill Builder

Side Lengths and Areas of Squares

The side length and area of a square are related.

- The area is the **square** of the side length.

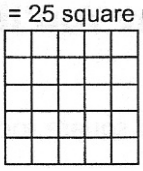


$$\begin{aligned}\text{Area} &= (\text{Length})^2 \\ &= 5^2 \\ &= 5 \times 5 \\ &= 25\end{aligned}$$

The area is 25 square units.

- The side length is the **square root** of the area.

Area = 25 square units

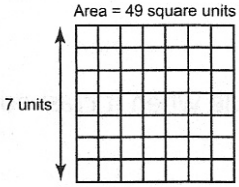
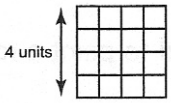
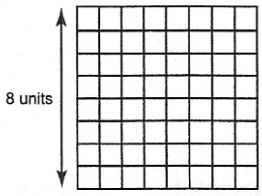
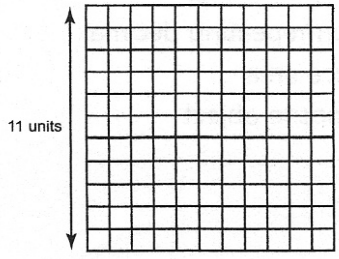


$$\begin{aligned}\text{Length} &= \sqrt{\text{Area}} \\ &= \sqrt{25} \\ &= \sqrt{5 \times 5} \\ &= 5\end{aligned}$$

The side length is 5 units.

Check

1. Which square and square root are modelled by each diagram?

Diagram	Square Modelled	Square Root Modelled
a) Area = 49 square units 	$(\text{Length})^2 = \text{Area}$ $7^2 =$ _____ The area is 49 square units.	$\sqrt{\text{Area}} = \text{Length}$ $\sqrt{49} =$ _____ The side length is 7 units.
b) 	_____ = _____ The area is _____ square units.	$\sqrt{\text{_____}} =$ _____ The side length is _____ units.
c) 	_____ = _____ The area is _____ square units.	$\sqrt{\text{_____}} =$ _____ The side length is _____ units.
d) 	_____ = _____ The area is _____ square units.	$\sqrt{\text{_____}} =$ _____ The side length is _____ units.

Whole Number Squares and Square Roots

- The square of a number is the number multiplied by itself.
- A square root of a number is one of 2 equal factors of the number.
- Squaring and taking a square root are inverse operations.

$5^2 = 5 \times 5$
$= 25$
$\sqrt{25} = \sqrt{5 \times 5}$
$= 5$
$5^2 = 25$ and $\sqrt{25} = 5$

Check

1. Complete each sentence.

a) $4^2 = 16$, so $\sqrt{16} =$ ____

b) $12^2 =$ ____, so $\sqrt{\quad} =$ ____

c) $\sqrt{25} =$ ____, since ____ $= 25$

d) $\sqrt{100} =$ ____, since ____ $=$ ____

Perfect Squares

A number is a **perfect square** if it is the product of 2 equal factors.
25 is a perfect square because $25 = 5 \times 5$.
24 is a **non-perfect square.** It is not the product of 2 equal factors.

Check

1. Complete each sentence.

First 12 Whole-Number Perfect Squares			
Perfect Square	**Square Root**	**Perfect Square**	**Square Root**
$1^2 = 1 \times 1 = 1$	$\sqrt{1} = 1$	$7^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\quad} =$ ___
$2^2 = 2 \times 2 = 4$	$\sqrt{4} = 2$	$8^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\quad} =$ ___
$3^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\quad} =$ ___	$9^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\quad} =$ ___
$4^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\quad} =$ ___	$10^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\quad} =$ ___
$5^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\quad} =$ ___	$11^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\quad} =$ ___
$6^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\quad} =$ ___	$12^2 =$ ___ $\times$ ___ $=$ ___	$\sqrt{\quad} =$ ___

1.1 Square Roots of Perfect Squares

FOCUS **Find the square roots of decimals and fractions that are perfect squares.**

The square of a fraction or decimal is the number multiplied by itself.

$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3}$

$= \frac{2 \times 2}{3 \times 3}$

$= \frac{4}{9}$

$(1.5)^2 = 1.5 \times 1.5$

$= 2.25$

$\frac{4}{9}$ and 2.25 are perfect squares because they are the product of 2 equal factors.

$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, so

$\frac{2}{3}$ is a square root of $\frac{4}{9}$.

We write: $\sqrt{\frac{4}{9}} = \frac{2}{3}$

$2.25 = 1.5 \times 1.5$, so

1.5 is a square root of 2.25.

We write: $\sqrt{2.25} = 1.5$

Each equal factor is a square root of the perfect square.

Example 1 Finding a Perfect Square Given Its Square Root

Calculate the number whose square root is:

a) $\frac{5}{8}$ **b)** 1.2

Solution

A square root of a number is one of two equal factors of the number.

a) $\frac{5}{8}$

$\frac{5}{8} \times \frac{5}{8} = \frac{5 \times 5}{8 \times 8}$

$= \frac{25}{64}$

So, $\frac{5}{8}$ is a square root of $\frac{25}{64}$.

b) 1.2

Use a calculator.

$1.2 \times 1.2 = 1.44$

So, 1.2 is a square root of 1.44.

Check

1. Calculate the perfect square with the given square root.

a) $\frac{3}{8}$

$\frac{3}{8} \times \frac{3}{8} = \frac{\_\_\_ \times \_\_\_}{\_\_\_ \times \_\_\_}$

$= \frac{\_\_\_}{\_\_\_}$

$\frac{3}{8}$ is a square root of _____.

b) $\frac{3}{2}$

$\_\_\_ \times \_\_\_ = \frac{\_\_\_}{\_\_\_}$

$= \frac{\_\_\_}{\_\_\_}$

$\frac{3}{2}$ is a square root of _____.

c) 0.5

$0.5 \times 0.5 =$ _____

0.5 is a square root of _____.

d) 2.5

$2.5 \times 2.5 =$ _____

2.5 is a square root of _____.

Example 2 Identifying Fractions that Are Perfect Squares

Is each fraction a perfect square? If so, find its square root.

a) $\frac{16}{25}$

b) $\frac{9}{20}$

Solution

Check if the numerator and denominator are perfect squares.

a) $\frac{16}{25}$

$16 = 4 \times 4$, so 16 is a perfect square.

$25 = 5 \times 5$, so 25 is a perfect square.

So, $\frac{16}{25}$ is a perfect square.

b) $\frac{9}{20}$

$9 = 3 \times 3$, so 9 is a perfect square.

20 is not a perfect square.

So, $\frac{9}{20}$ is not a perfect square.

Check

1. Determine whether the fraction is or is not a perfect square. How do you know?

a) $\frac{9}{49}$

9 _____ a perfect square because _____.

49 _____ a perfect square because _____.

So, $\frac{9}{49}$ _____ a perfect square.

b) $\frac{25}{13}$

25 _____ a perfect square because _____.

13 _____ a perfect square because _____.

So, $\frac{25}{13}$ _____ a perfect square.

c) $\frac{64}{81}$ 64 _____ a perfect square because _____.

81 _____ a perfect square because _____.

So, $\frac{64}{81}$ _____ a perfect square.

2. Find the value of each square root.

a) $\sqrt{\frac{9}{4}} = \sqrt{\frac{\times}{\times}} = \frac{\quad}{\quad}$

b) $\sqrt{\frac{16}{81}} = \sqrt{\frac{\times}{\times}} = \frac{\quad}{\quad}$

A **terminating decimal** ends after a certain number of decimal places.
A **repeating decimal** has a repeating pattern of digits in the decimal expansion.
The bar shows the digits that repeat.

Terminating	Repeating	Non-terminating and non-repeating
0.5 0.28	$0.333\,333\,\ldots = 0.\overline{3}$	1.414 213 56 ... 7.071 067 812 ...
	$0.191\,919\,\ldots = 0.\overline{19}$	

You can use a calculator to find out if a decimal is a perfect square.
The square root of a perfect square decimal is either a terminating decimal or a repeating decimal.

Example 3 Identifying Decimals that Are Perfect Squares

Is each decimal a perfect square? How do you know?

a) 1.69 **b)** 3.5

Solution

Use a calculator to find the square root of each number.

a) $\sqrt{1.69} = 1.3$

The square root is the terminating decimal 1.3.
So, 1.69 is a perfect square.

b) $\sqrt{3.5} \doteq 1.870\,828\,693$

The square root appears to be a decimal that neither repeats nor terminates.
So, 3.5 is not a perfect square.

The symbol $\doteq$ means "approximately equal to".

Check

1. Complete the table to find whether each decimal is a perfect square. The first one is done for you.

	Decimal	Value of square root	Type of decimal	Is decimal a perfect square?
a)	70.5	8.396 427 811 ...	Non-repeating Non-terminating	No
b)	5.76	_____	_____ _____	_____
c)	0.25	_____	_____ _____	_____
d)	2.5	_____	_____ _____	_____

Practice

1. Calculate the number whose square root is:

a) $\frac{1}{4}$

$\frac{1}{4} \times \frac{1}{4} = \frac{\_\_ \times \_\_}{\_\_ \times \_\_}$

$= \frac{\_\_}{\_\_}$

$\frac{1}{4}$ is a square root of ____.

b) $\frac{2}{7}$

$\_\_ \times \_\_ = \frac{\_\_ \times \_\_}{\_\_ \times \_\_}$

$= \frac{\_\_}{\_\_}$

$\frac{2}{7}$ is a square root of ____.

c) 0.6

_____ × _____ = _____

0.6 is a square root of _____.

d) 1.1

_____ × _____ = _____

1.1 is a square root of _____.

2. Identify the fractions that are perfect squares. The first one has been done for you.

	Fraction	Is numerator a perfect square?	Is denominator a perfect square?	Is fraction a perfect square?
a)	$\frac{81}{125}$	Yes; $9 \times 9 = 81$	No	No
b)	$\frac{25}{49}$	_____	_____	_____
c)	$\frac{36}{121}$	_____	_____	_____
d)	$\frac{17}{25}$	_____	_____	_____
e)	$\frac{9}{100}$	_____	_____	_____

3. Find each square root.

a) $\sqrt{\frac{49}{100}} = \sqrt{\frac{\square \times \square}{\square \times \square}}$

$= \frac{\square}{\square}$

b) $\sqrt{\frac{25}{144}} = \sqrt{\frac{\square \times \square}{\square \times \square}}$

$= \frac{\square}{\square}$

c) $\sqrt{\frac{1}{16}} = \sqrt{\frac{\square \times \square}{\square \times \square}}$

$= \frac{\square}{\square}$

d) $\sqrt{\frac{9}{400}} = \sqrt{\frac{\square \times \square}{\square \times \square}}$

$= \frac{\square}{\square}$

4. Use a calculator. Find each square root.

a) $\sqrt{8.41} =$ _____ **b)** $\sqrt{0.0676} =$ _____ **c)** $\sqrt{51.125} =$ _____ **d)** $\sqrt{6.25} =$ _____

5. Which decimals are perfect squares?

a) 1.44 $\sqrt{1.44} =$ _____

The square root is a decimal that _____.

So, 1.44 _____ a perfect square.

b) 30.25 $\sqrt{30.25} =$ _____

The square root is a decimal that _____.

So, 30.25 _____ a perfect square.

c) 8.5 $\sqrt{8.5} \doteq$ _____

The square root is a decimal that _____.

So, 8.5 _____ a perfect square.

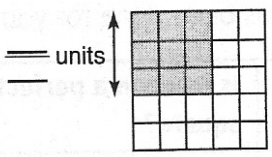
d) 0.0256 $\sqrt{0.0256} =$ _____

The square root is a decimal that _____.

So, 0.0256 _____ a perfect square.

6. Find the area of each square.

a)

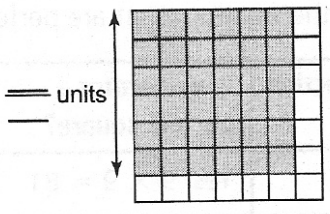


Area = _____

$= \frac{\square}{\square}$

$= \frac{\square}{\square}$

b)



Area = _____

$= \frac{\square}{\square}$

$= \frac{\square}{\square}$

Area = $(\text{Length})^2$

The area is _____ _____

c) 5.4 units

Area = ____

= ____ × ____

= ____

d) 2.1 units

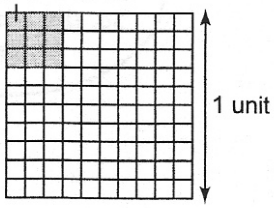
Area = ____

= ____ × ____

= ____

7. Find the side length of each square.

a) Area = $\frac{9}{100}$ square units



Side Length = $\sqrt{}$ ____

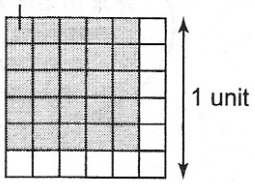
= $\sqrt{}$ ____

= ____

Length = $\sqrt{\text{Area}}$

The side length is ___ units.

b) Area = $\frac{25}{36}$ square units

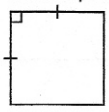


Length = $\sqrt{}$ ____

= $\sqrt{}$ ____

= ____

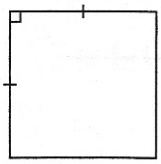
c) Area = 0.01 square units



Length = $\sqrt{}$ ____

= ____

d) Area = 46.24 square units



Length = $\sqrt{}$ ____

= ____