

Lesson #1 – Definitions

Use the glossary at the back of "Math Makes Sense" to define the following. Provide examples if you are able.

1. Variable:

- Represented by a letter
- A quantity that can vary

2. Polynomial: → HIGHEST EXPONENT TO LOWEST EXPONENT

- Made up of Terms separated by + or - signs.
- $\sqrt{\quad}$: square roots and variables in denominators are NOT polynomials.

3. Terms:

- Make up polynomials
- Made up of a variable, a number and a whole number exponent

4. Coefficient:

- Can be positive or negative
- The numerical factor of a term → the number with the variable!

5. Constants:

- The number that does not change
- It is the number not attached to a variable
- ↳ (In reality, it is attached to the variable to the 0 exponent)

6. Degree:

- the value of the highest exponent of a term (polynomial)
- the value of the exponent (term)

7. Monomial:

- Mono → 1
- polynomial made up of 1 term

8. Binomial:

- Bi → 2
- polynomial made up of 2 terms

9. Trinomial:

- Tri → 3
- polynomial made up of 3 terms

Classifying the degree of a Monomial, Binomial and Polynomial:

Classify the degree of the following monomials:

a) $5x$

x is x^1

1 Degree

b) $10x^2$

2 Degree

c) x^2y

2 Degree

Monomials have a degree of the number of the exponent on all variables. If there is only one variable, the degree is the exponent

**If there is a variable by itself it is raised to the power of 1

ie: x

A **constant term always has a degree of 0

ie: 5

$5x^0$ $x^0 = 1$ so $5x^0 = 5 \cdot 1 = 5$

Classify the degree of the following. Indicate if it is a binomial, trinomial or polynomial.

a) $3x^2 + 2x + 1$

2 Degree

Trinomial

b) $x^2y^3 + y^4$

4 Degree

Binomial

c) $2x^4 + x^3 - 5x^2 + 3x - 7$

4 Degree

Polynomial

A polynomial has the degree of the largest value of the exponents. The single term with the highest sum of exponents is the degree of the entire polynomial.

Ex) Complete the table below:

Polynomial	Variable(s)	Degree	Number of Terms	Coefficient(s)	Constant
$3x^2 + 4x - 2$	x	<u>2</u>	<u>3</u>	<u>3, 4</u>	<u>-2</u>
$-4x^2y^3 + 2y^2 + 3y$	x, y	<u>3</u>	<u>3</u>	<u>-4, 2, 3</u>	<u>0</u>
<u>$4x^3 - 2$</u>	x	<u>3</u>	<u>2</u>	<u>4</u>	<u>-2</u>
$5x^5y + 4x^2 - 13$	x, y	<u>5</u>	<u>3</u>	<u>5, 4</u>	<u>-13</u>
<u>$4x + 3$</u>	x	<u>1</u>	<u>2</u>	<u>4</u>	<u>3</u>

Writing Polynomials:

When you create a polynomial from some given terms it is important to understand there is not one single correct answer. There are many, many correct answers, however to be correct every stipulation outlined must hold true.

For consistency when writing polynomials there are a few conditions that must be met:

1. Terms are written from the highest degree to the lowest degree (descending order).

$x^3 + 5x^2 - 11x + 5$ is written correctly, while

$5x^2 + 5 - 11x + x^3$ is written **incorrectly**.

2. If the degree between two terms is the same, then the variables are written alphabetically.

$x^3 - y^3 + 2x^2 + y^2 - 3x + 5y - 11$ is written correctly, while

$-y^3 + x^3 + y^2 + 2x^2 - 3x + 5y - 11$ is written **incorrectly**.

3. The constant always goes at the end (which follows the logic of the degree being in descending order, since a constant has a degree of _____).

A **polynomial** must have **whole number exponents** on the variable. So, variables may be raised to the powers 0, 1, 2, 3, This means that any expression where the variable is in the denominator ($\frac{1}{x}$) and would therefore have a negative exponent {think back to the powers unit - ($\frac{1}{x} = (\frac{x}{1})^{-1} = x^{-1}$)} is **not a polynomial**. Also, any expression where there is a square root of the variable ($\sqrt{x} = x^{\frac{1}{2}}$) does not have a **whole number exponent** and is therefore not classified as a polynomial.

Ex) Re-write the following polynomials according to the conventions above. If the expression is not a polynomial, state so.

a) $2x^2 - 5x^3 + 8 - x$

Polynomial

$-5x^3 + 2x^2 - x + 8$

c) $\sqrt{x} - 11$

Not a polynomial

b) $\frac{x}{3} + 5x^2 - 11$

Polynomial

$5x^2 + \frac{x}{3} - 11$

d) $\frac{3}{x} - 4x^3 - 11 + 2x^2$

Not a polynomial

e) $2 - 3x + 11x^2 - 3x^3$

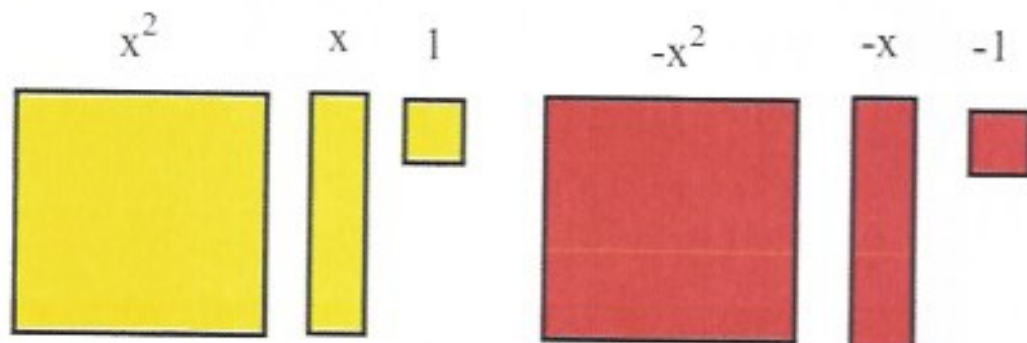
$-3x^3 + 11x^2 - 3x + 2$

Assignment: Page 214 #4-7, 9, 10.

Lesson #2 – Modeling Polynomials

Algebra Tiles (or Alge-tiles) are used to represent integers and variables. Lighter colored tiles (yellow or white on paper) represent positive numbers or variables, while darker colored tiles (red or grey on paper) represent negative numbers or variables.

Ex:



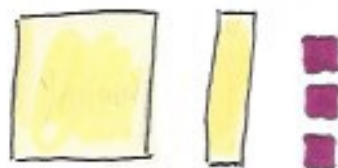
Alge-tiles can be used to represent polynomial expressions, for example the expression $x^2 - 3x + 5$ has the individual terms of a positive x^2 , three negatives x 's and 5 ones. This would be represented by:



The different exponents on the variable indicate which tile needs to be used, while the **coefficient** indicates how many of each tile should be used. When you draw your own you have two options, you may shade the tiles for negatives or include positive and negative symbols on each to show their sign.

Ex) Model each expression by sketching the algetiles used to represent it.

a) $x^2 + x - 3$



b) $-2x^2 - 3$



c) $2x^2 + 3$



d) $-2x^2 - 3x + 1$



e) $-3x + 3$



Ex2) Write the polynomial that each set of algebra tiles represents.

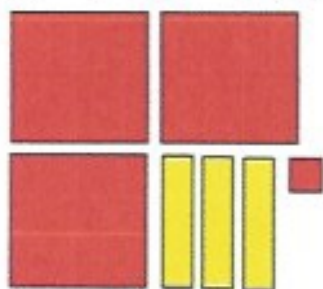
a) $x^2 - x + 3$



d) $2x - x^2 - 5 \rightarrow -x^2 + 2x - 5$



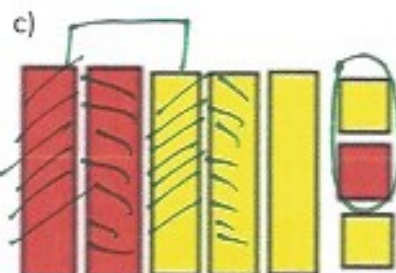
b) $-3x^2 + 3x - 1$



e)



$2x^2 + 2x$



Make zero pairs
 $x + 1$

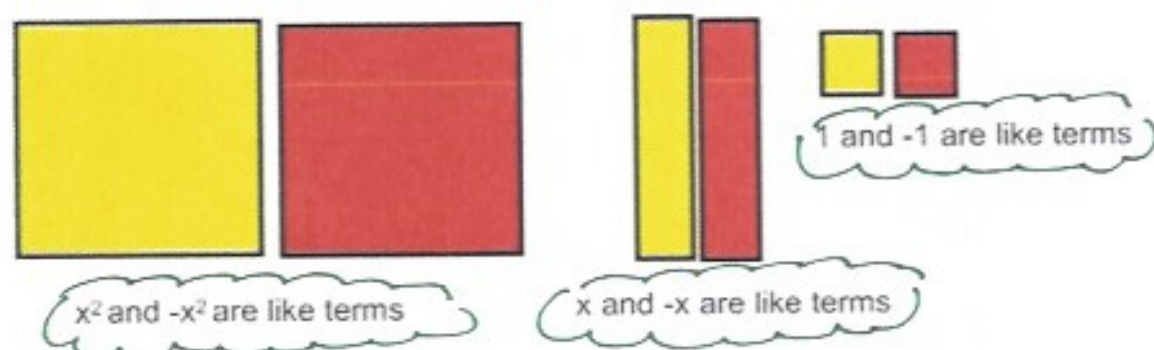
Assignment: Page 214 #8, 11, 12, 13

{ Make zero pairs }

Lesson #3 – Like and Unlike Terms

Remember the idea of a zero pair. For instance 3 and -3 are zero pairs because their sum is zero. The same is true for some algebra tiles.

When working with algebra we can only group **like terms**. These are the terms that can be modelled by the same tiles:



Like terms have Same variable and the Same exponent
 Examples of like terms are: (coefficient does not determine equality)
 $3x^2$ and $-4x^2$ $3n$ and $-0.75n$

Un-alike terms have Different variables and Different exponents

Examples of unlike terms are:

$$7m^2 \text{ and } -6m \text{ or } 3y^3$$

Ex) Identify the like terms in each expression.

a) $2x + 3y - 4xy + 5x - 2y + xy$

$-4xy$ and xy
 $2x$ and $5x$
 $3y$ and $-2y$

b) $2a + 5a - 6b + 8b - 2c + 3c$

✓
like ✓
like ✓
like

c) $3s^2 + 5s - 2 + 7$

✓
like

The co-efficient on the variable shows you **quantity** (like how many you have).

Like terms can be grouped. Unlike terms must remain separate.

→ You can only "group" like terms !!!

You can use algebriles to simplify polynomials that are not in simplest form.

If you are asked to simplify a polynomial it means you are being asked to combine any terms that are like.

Ex) To simplify $2x^2 + 3x - 5 + x^2 - 2x + 3$ using algebriles:

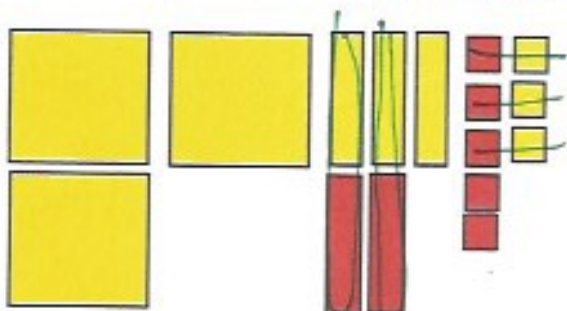
- First you would need to sketch all algebriles you need to use.



Simplify

group like

- Second you will want to group all alike tiles



terms

$$(2x^2 + 3x - 5 + x^2 - 2x + 3)$$

$$\rightarrow 2x^2 + x^2 + 3x - 2x - 5 + 3$$

$$\rightarrow 3x^2 + x - 2$$

Tip: **Line up zero pairs**

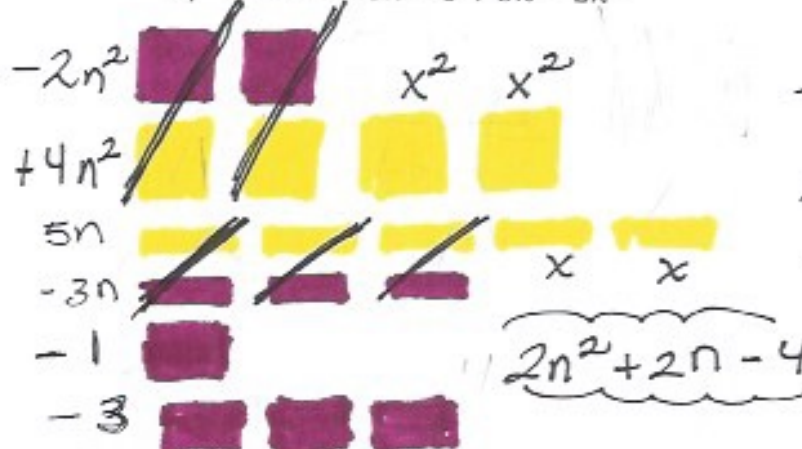
- Finally, count up how many tiles you have; count every zero pair as none.

This will tell you that $2x^2 + 3x - 5 + x^2 - 2x + 3 = 3x^2 + x - 2$

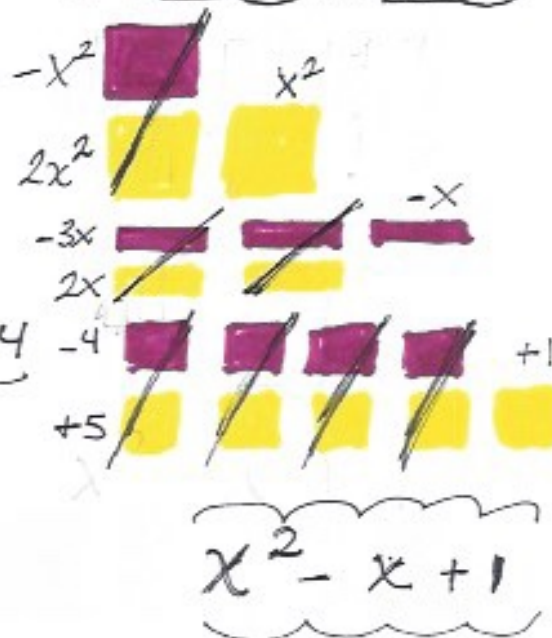
A simpler way of doing this is writing all like terms in groups in the first step.

Ex) Draw algebriles to simplify the following:

a) $4n^2 - 1 - 3n - 3 + 5n - 2n^2$



b) $-x^2 + 5 - 3x + 2x^2 - 4 + 2x$



Lesson #4 – Adding Polynomials

5.3

There are two methods for adding polynomials. They both require an understanding of like terms which are terms with the same variable raised to the same degree.

Using Alge-tiles to Determine a Sum:

Method 1

One method which is very effective to determine the sum of a polynomial is using algetiles.

This is an effective method but is only practical if the coefficients are smaller numbers.

Ex) Calculate the sum of $(3x^2 + 2x + 4) + (-x^2 + 3x - 5)$

- Each set of brackets represents a separate polynomial. Display each polynomial using the appropriate tile:

$$(3x^2 + 2x + 4)$$



$$(-x^2 + 3x - 5)$$



- Combine the displays and group like tiles:



- Remove zero pairs

The remaining tiles represent the sum.



$$2x^2 + 5x - 1$$

Grouping Like terms to solve Algebraically:

Like terms can be grouped symbolically to add a polynomial as well. This process is much more logical if the coefficients are larger numbers.

Ex) Calculate the sum of $(15n^2 - 17n + 24) + (11n^2 - 21n + 15)$

- Each set of brackets still represents a separate polynomial, but this method requires us to use the property of distribution to rid ourselves of the brackets. Image that each set is being multiplied by 1 (because there is no number out front and no number in math is always an invisible 1). This will allow us to write the polynomial with no brackets:

$$15n^2 - 17n + 24 + 11n^2 - 21n + 15$$

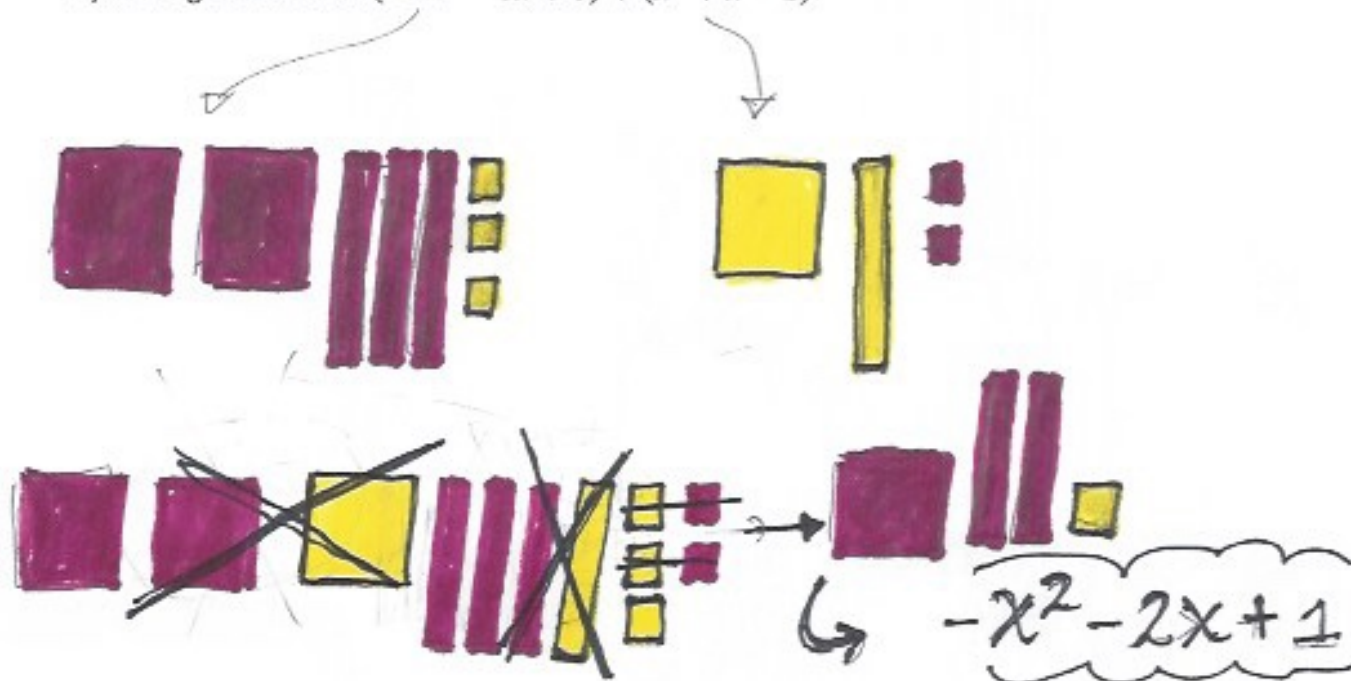
- Now rearrange the terms so that all like terms are together. Be aware of the operator before the term! The proper operator needs to accompany each term.

$$15n^2 + 11n^2 - 17n - 21n + 24 + 15$$

- Finally, add or subtract each coefficient to combine like terms.

$$26n^2 - 38n + 39$$

Ex) Use alge tiles to add $(-2x^2 - 3x + 3) + (x^2 + x - 2)$



Ex) Add the following algebraically: $(22x^2 + 17x - 35) + (11x^2 + 12x - 17)$

Remove the bracket

$$22x^2 + 17x - 35 + 11x^2 + 12x - 17$$

$$\underbrace{22x^2 + 11x^2} + \underbrace{17x + 12x} - \underbrace{35 - 17}$$

↓ ↓ ↓

$33x^2 + 29x - 52$

Sometimes polynomials may have more than one variable in them. In this case like terms are still defined as having the same variable(s) to the same degree.

Ex) Add the following: $(2a^2 + a - 3b - 7ab + 3b^2) + (-4b^2 + 3ab + 6b - 5a + 5a^2)$

Remove Bracket

$$2a^2 + a - 3b - 7ab + 3b^2 - 4b^2 + 3ab + 6b - 5a + 5a^2$$

Group Like terms

$$\underbrace{2a^2 + 5a^2} + \underbrace{3b^2 - 4b^2} + \underbrace{a - 5a} - \underbrace{3b + 6b} - 7ab + 3ab$$

$$7a^2 - b^2 - 4a + 3b - 4ab$$

$$\therefore 7a^2 - b^2 - 4a + 3b - 4ab$$

5.4

Lesson #5 – Subtracting Polynomials

to SUBTRACT
↳ YOU ADD THE
OPPOSITE

Polynomials can be subtracted both algebraically and by using algetiles along with the **additive inverse**. The additive inverse is the expression that would add to zero ; for example the additive inverse of 3 is -3 because $3 + (-3) = 0$.

To subtract polynomials using algetiles you would adding the **inverse polynomial**.

State the **additive inverse** of the polynomials below:

- $x^2 + 2x + 3$ would be represented by:



So

$$x^2 + 2x + 3$$

So the inverse would be all the opposite:



inverse

$$-x^2 - 2x - 3$$

$$(-x^2 - 2x - 3)$$

Together these would add to zero.

Ex) State the additive inverse to the following:

a) $2x^2 - 3x + 1$

↓ ↓ ↓
 $-2x^2 + 3x - 1$

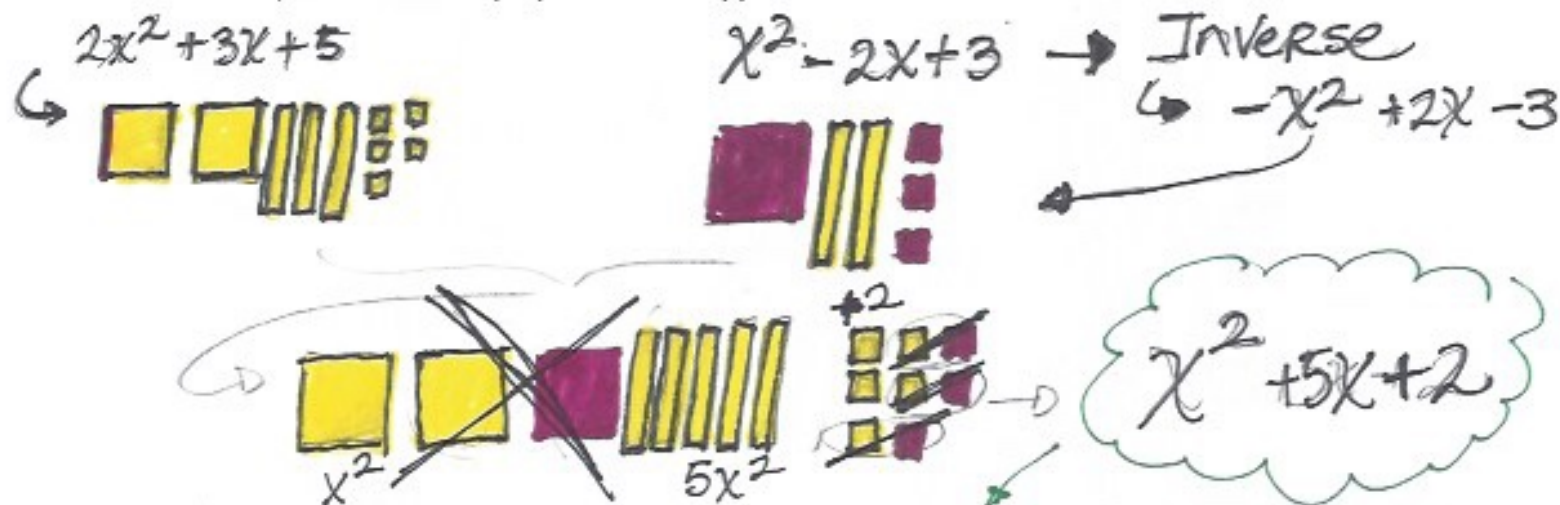
b) $-3x^2 + 2x - 6$

↓ ↓ ↓
 $3x^2 - 2x + 6$

Method 1 – Subtract the following polynomial by using algebra tiles and the additive inverse.

$$(2x^2 + 3x + 5) - (x^2 - 2x + 3)$$

- Represent the first polynomial as it appears.



- To subtract the second polynomial use the additive inverse of it. This means you will add the algebra tiles that would represent the additive inverse.
- Line up any zero pairs to eliminate them and state what is left symbolically as you did with adding.

$$\therefore (2x^2 + 3x + 5) - (x^2 - 2x + 3) = 2x^2 + 3x + 5 - x^2 + 2x - 3$$

$$2x^2 - x^2 + 3x + 2x + 5 - 3 = x^2 + 5x + 2$$

Method 2 – Subtract the following polynomials using the concept of distribution of the negative over the second polynomial.

$$(3x^2 - 2x + 4) - (2x^2 + x - 2)$$

- Distribute the negative sign over the second polynomial. This will make it the additive inverse of the original expression. It is easier to see if you re-write the expression as

$$(3x^2 - 2x + 4) + (-1)(2x^2 + x - 2)$$

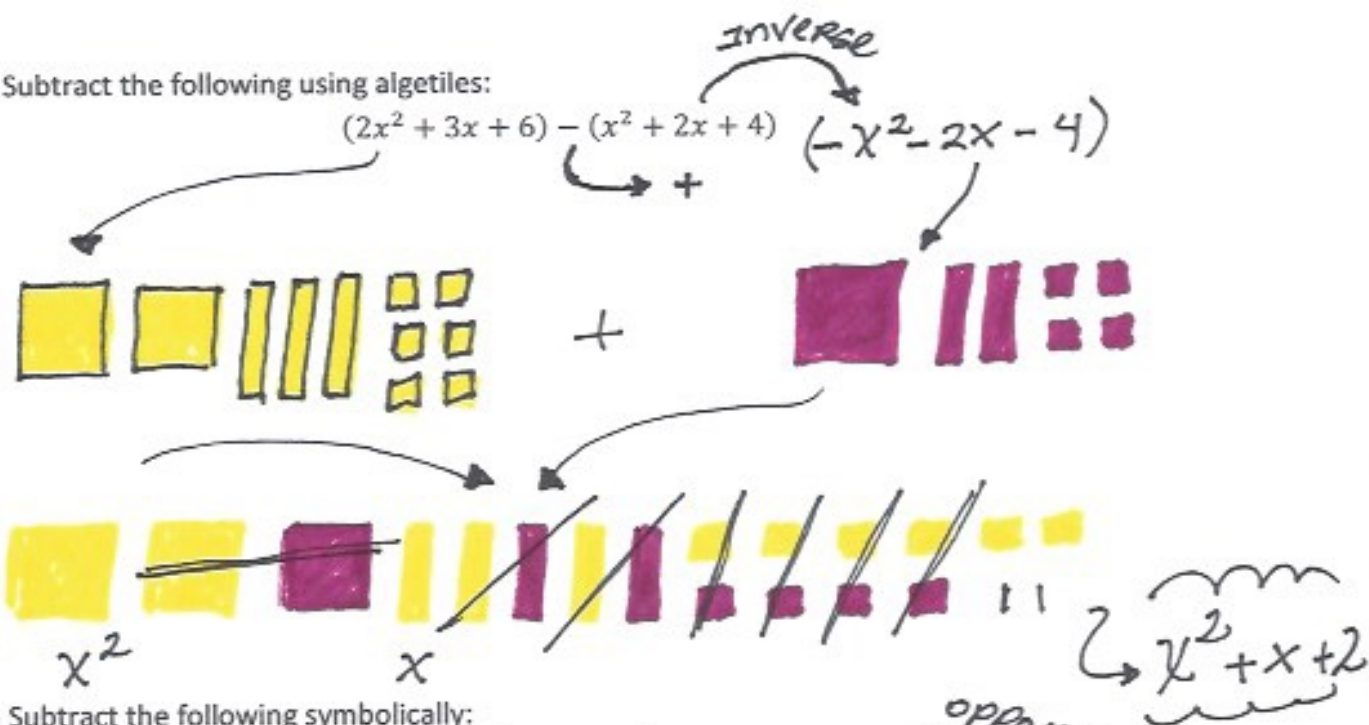
$$3x^2 - 2x + 4 - 2x^2 - x + 2$$

$$3x^2 - 2x^2 - 2x - x + 4 + 2$$

$$x^2 - 3x + 6$$

- Now you should have two polynomials to add, which you will do as you did in the adding polynomials section by adding like terms with the same variable to the same degree.
- Another way to think of this is that you will be adding the additive inverse to the first polynomial.

Ex 1) Subtract the following using algetiles:



Ex 2) Subtract the following symbolically:

$$\begin{aligned}
 & (13x^2 + 3x + 5y + 6y^2) - (11x^2 - 12x + 7y + 14y^2) \\
 & = (13x^2 + 3x + 5y + 6y^2) + (-11x^2 + 12x - 7y - 14y^2) \\
 & = \boxed{13x^2} + \boxed{3x} + \boxed{5y} + \boxed{6y^2} - \boxed{11x^2} + \boxed{12x} - \boxed{7y} - \boxed{14y^2} \\
 & = \underline{13x^2 - 11x^2} + \underline{6y^2 - 14y^2} + \underline{3x + 12x} + \underline{5y - 7y} \\
 & = 2x^2 - 8y^2 + 15x - 2y
 \end{aligned}$$

Lesson #6 – Multiplying Polynomials by a Constant

Recall that a constant is any number with no variable attached to it; also recall multiplication by grouping that you learned when first learning multiplication tables. So, if you were to multiply $(2)(3)$ you would represent 2 groupings of 3:



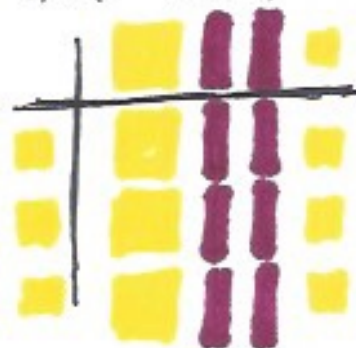
This rectangle is 2 units by 3 units and represents $(2)(3)$ being equivalent to 6.

The same can be done when multiplying any polynomial by a constant. For example, $2(x - 1)$ would be represented by making two groups of $(x - 1)$:

This would mean that $2(x - 1) = 2x - 2$

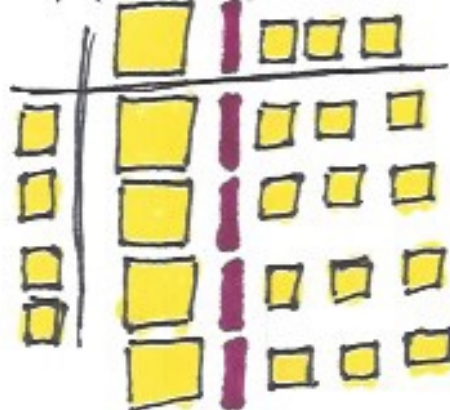
Method 1 – Multiply the following by showing the groupings with algetiles:

a) $3(x^2 - 2x + 1)$



$\hookrightarrow 3x^2 - 6x + 3$

b) $4(x^2 - x + 3)$



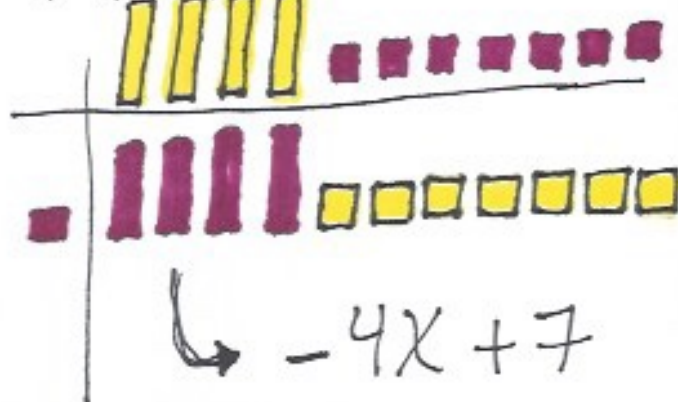
$\hookrightarrow 4x^2 - 4x + 12$

c) $-2(x^2 + 3x)$



$\hookrightarrow -2x^2 - 6x$

d) $-1(4x - 7)$




$\hookrightarrow -4x + 7$

Method 2 - Multiply the following by using **distribution of the constant**.


When you multiply using distribution of the constant you are multiplying every term of the polynomial by that constant. This is done by multiplying the coefficients as though they were just numbers and keeping the variable and constant with them. When you are multiplying **like terms do not affect which terms are multiplied**. Every term will be multiplied by every other term.

Ex) Multiply the following:


a) $2(3x^2 + 7x + 1)$


$$(2 \cdot 3x^2) + (2 \cdot 7x) + (2 \cdot 1)$$
$$6x^2 + 14x + 2$$

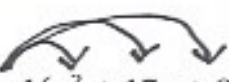
c) $6(-4x^3 + 7xy + 5y^2)$


$$-24x^3 + 42xy + 30y^2$$

b) $-3(5x^2 - 11x + 6)$


$$-15x^2 + 33x - 18$$

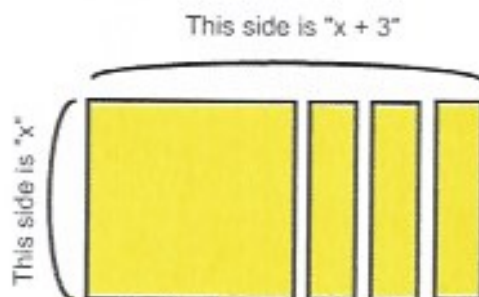
d) $-1(x^2 + 17x + 8)$


$$-x^2 - 17x - 8$$

Lesson #7 – Multiplying a Polynomial by a Monomial

Recall that a **monomial** is a polynomial with one term. This means you are multiplying one term with a variable by a multi-term polynomial with variables as well as constants. This can be done with alge tiles to illustrate how it would look.

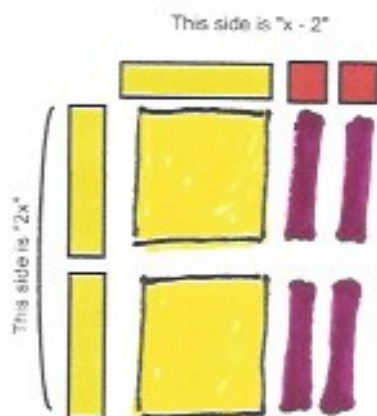
Consider the expression $x(x + 3)$. You can think of it like making “ x ” groupings of “ $(x + 3)$ ” which is difficult to consider. Another way is to think that you are making a rectangle with the dimensions of “ x ” groupings of “ $(x + 3)$ ”:



This would mean that the product of $x(x + 3) = x^2 + 3x$

This is easier to create by using the monomial and polynomial you are multiplying as outside dimensions of a rectangle, then filling the inside in.

Ex) a) Multiply $2x(x - 2)$

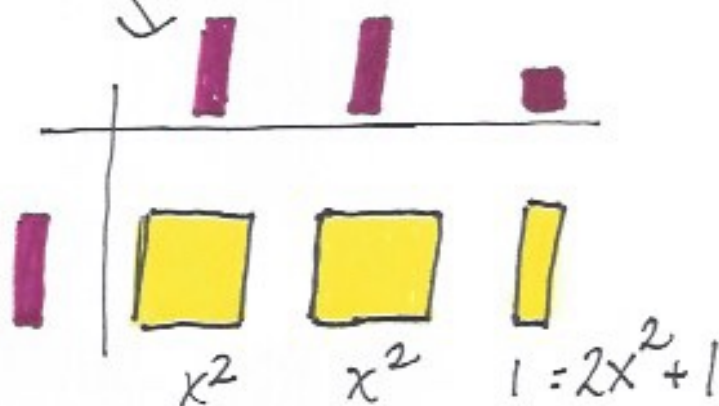
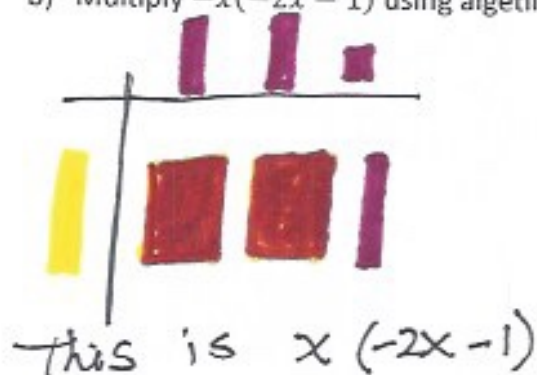


➤ Use “ $2x$ ” and “ $(x - 2)$ ” as the outside dimensions. This requires a knowledge of integer multiplication to fill in the inside.

➤ Fill in the inside of these dimensions. Use your rules of integer multiplication; when you have a negative multiplied by a positive that would indicate a **negative**; etc.

➤ Using **just the inside that you have filled in**, count the alge tiles you have drawn and write them symbolically to indicate the product.

b) Multiply $-x(-2x - 1)$ using alge tiles.



Numbers with numbers
Variables with variables

Method 2 - Multiplication of a polynomial by a monomial algebraically (symbolically). When you multiply a polynomial by a polynomial you must recall your exponent laws. For example, you know that $2^3 \cdot 2^5 = 2^8$. The same would be true for any variables that are being multiplied; so $x^3 \cdot x^5 = x^8$. When you multiply a monomial by a polynomial every term within the polynomial must be multiplied by the monomial. To do this you must multiply the coefficient by the coefficient and **add the exponents** on the variable. Remember that a singular variable (like x would have an exponent of 1!).

This method is best used when you have more than one variable or the exponents will be larger than 2.

Ex) Multiply the following:

a) $2x(3x - 5)$

$$\begin{aligned} & (2x \cdot 3x) - (5 \cdot 2x) \\ & 6x^{1+1} - 10x \\ & 6x^2 - 10x \end{aligned}$$

b) $2x(x^2 + 2x + 4)$

$$\begin{aligned} & (2x \cdot x^2) + (2x \cdot 2x) + (2x \cdot 4) \\ & 2x^{1+2} + 4x^{1+1} + 8x \\ & 2x^3 + 4x^2 + 8x \end{aligned}$$

c) $11x(2x^2 - 5x + 3)$

$$\begin{aligned} & (11x \cdot 2x^2) - (11x \cdot 5x) + (11x \cdot 3) \\ & 22x^{1+2} - 55x^{1+1} + 33x \\ & 22x^3 - 55x^2 + 33x \end{aligned}$$

d) $-5x(3x^2 + 7x - 5)$

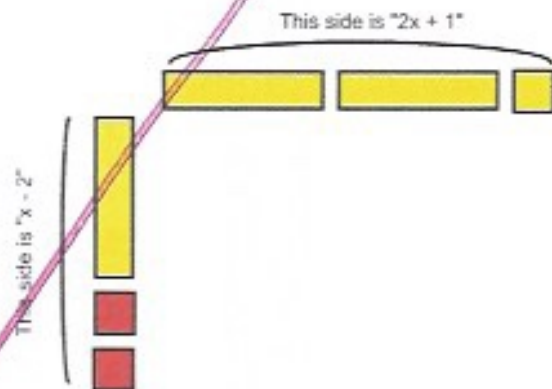
$$\begin{aligned} & (-5x \cdot 3x^2) + (-5x \cdot 7x) + (-5x \cdot -5) \\ & -15x^{1+2} - 35x^{1+1} + 25x \\ & -15x^3 - 35x^2 + 25x \end{aligned}$$

Lesson #8 – Multiplying a Polynomial by a Polynomial

Consider the multiplication of the polynomials $(x - 2)(2x + 1)$. These can be done with algetiles in much the same manner as a polynomial being multiplied by a monomial.

Method 1 – Using Algetiles

- Create a box with the outer dimensions of each polynomial
- Fill in the inside of the box, using the properties of integer multiplication
- Count the algetiles you are left with



Ex1) Multiply the following using algetiles:

a) $(x - 3)(3x - 1)$

b) $(-x - 3)(2x - 1)$

Method 2 – Algebraically: You can also multiply polynomials algebraically. This requires a systematic approach to the multiplication as every term within every polynomial must be multiplied by another. The acronym FOIL helps when you are multiplying a binomial by a binomial.

Ex) Multiply the following:

a) $(x + 5)(2x - 3)$

b) $(-2x + 3)(3x + 1)$

FOIL stands for:

F -

O -

I -

L -

When you multiply a polynomial by a binomial you must use a different approach to distribution and be very aware that you are multiplying every single term by every single term. This is called **mega-distribution** and will be used for any larger polynomial multiplication. You must also apply the rules for monomial by polynomial multiplication from last class.

Ex) Multiply $(3x + 2)(x^2 - 3x - 4)$

Assignment:

1. $(4x - 2)(4x + 5)$

2. $(x + 5)(2x - 2)$

3. $(2x - 2)(4x + 4)$

4. $(4x - 1)(2x - 3)$

5. $(2x + 5)(3x - 1)$

6. $(x + 3)(3x - 5)$

7. $(2x + 4)(4x - 5)$

8. $(3x + 3)(4x - 2)$

9. $(3x - 5)(4x - 3)$

10. $(2n + 1)(4n + 5)$

11. $(3x - 2)(5x + 4)$

12. $(5n - 5)(n + 5)$

13. $(2x + 4)(3x - 4)$

14. $(3x - 3)(2x - 2)$

15. $(x - 1)(2x - 4)$

16. $(2x - 2)(5x + 5)$

17. $(2x - 4)(x - 2)$

18. $(5n - 2)(3n - 5)$

19. $(5n - 1)(3n + 5)$

20. $(2n - 3)(n + 4)$

This can also be done algebraically:

Ex) a) $(8x + 12) \div 4$

➤ Set up the division to be a fraction

$$\frac{8x + 12}{4} = \frac{8x}{4} + \frac{12}{4} = \underbrace{2x + 3}$$

- Separate the terms (think the opposite of when you are adding or subtracting fractions)
➤ Divide each coefficient of the polynomial by the constant.

b) $(-3x^2 - 15x + 24) \div (-3)$

$$= \frac{-3x^2 - 15x + 24}{-3}$$

$$= \frac{-3x^2}{-3} + \frac{-15x}{-3} + \frac{24}{-3}$$

$$= \underbrace{x^2 + 5x - 8}$$



c) $\frac{18x^2 + 9x - 27}{9}$

$$= \frac{\cancel{18}x^2}{\cancel{9}} + \frac{\cancel{9}x}{\cancel{9}} + \frac{-\cancel{27}}{\cancel{9}}$$

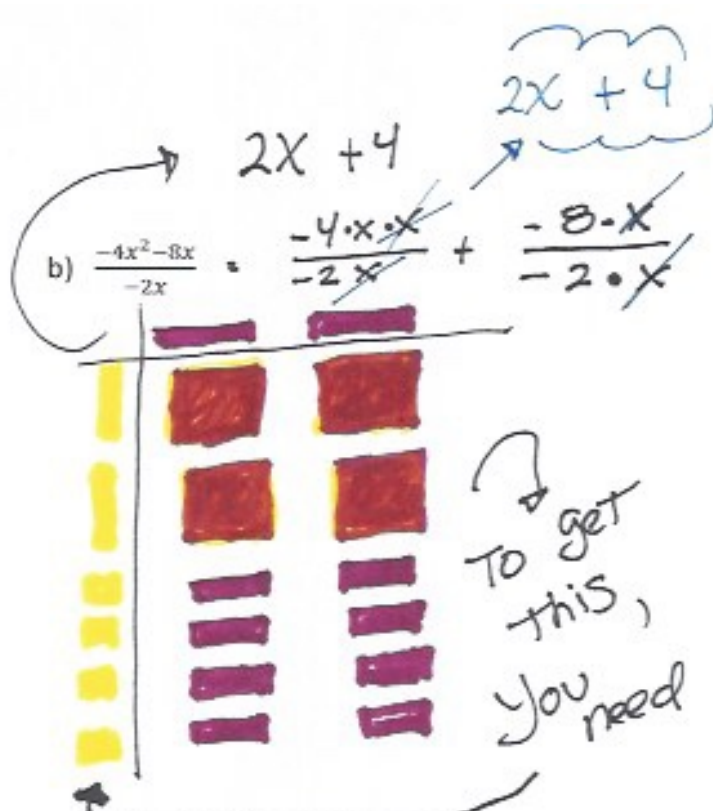
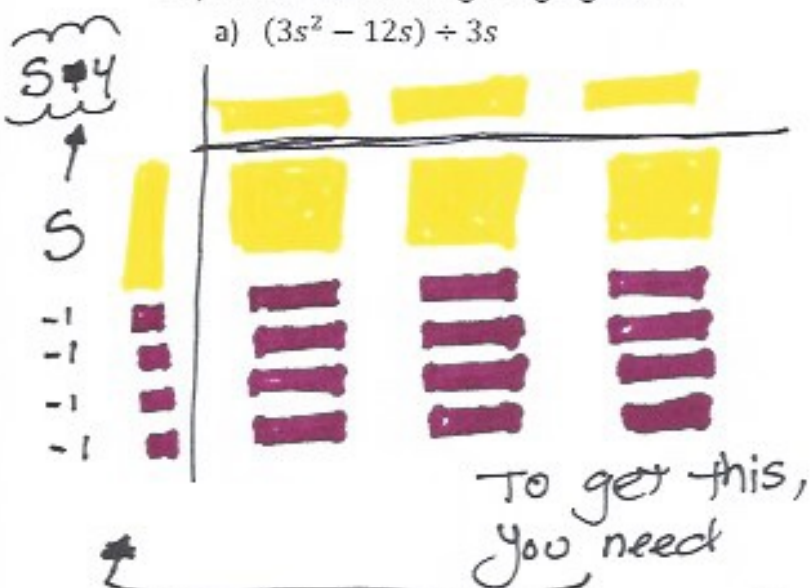
$$= \underbrace{2x^2 + x - 3}$$



Assignment: page 246-248 #6, 8, 13, 14, 16, 23

Ex) Divide the following using algetiles:

a) $(3s^2 - 12s) \div 3s$



You can also divide polynomials by monomials by using your exponent division laws. That is, you know that $\frac{3^8}{3^5} = 3^3$ because when the bases are the same and being divided you subtract your exponent. This would be true for variables also; that is $\frac{x^n}{x^5} = x^3$. When dividing algebraically you are not limited to how high your exponent goes nor are you limited to one variable.

Ex) Divide the following:

a) $(8x^2 + 14x) \div 2x$

$$= \frac{8x^2 + 14x}{2x}$$

$$= \frac{8 \cdot x \cdot x}{2 \cdot x} + \frac{14 \cdot x}{2 \cdot x}$$

$$= \frac{4x + 7}{1}$$

b) $\frac{-6y^3 + 24y^2 - 30y}{-6y}$

$$= \frac{-6 \cdot y \cdot y \cdot y}{-6y} + \frac{24 \cdot y \cdot y}{-6y} + \frac{-30 \cdot y}{-6y}$$

$$= y^2 - 4y + 5$$

c) $\frac{-9xy^3 + 18x^2y^2 - 27x^3y}{9xy}$

$$= \frac{-9 \cdot x \cdot y \cdot y \cdot y}{9 \cdot x \cdot y} + \frac{18 \cdot x \cdot x \cdot y \cdot y}{9 \cdot x \cdot y} + \frac{-27 \cdot x \cdot x \cdot x \cdot y}{9 \cdot x \cdot y}$$

$$= -y^2 + 2xy - 3x^2$$

d) $(15x^4 - 25x^3 + 40x^2 - 15x) \div 5x$

$$= \frac{15 \cdot x \cdot x \cdot x \cdot x}{5x} + \frac{-25 \cdot x \cdot x \cdot x}{5x} + \frac{40 \cdot x \cdot x}{5x} - \frac{15x}{5x}$$

$$= 3x^3 - 5x^2 + 8x - 3$$