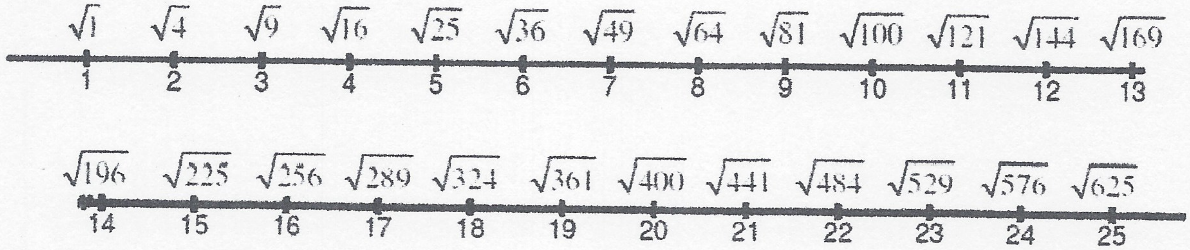
Section 1.2 – Square Roots and Perfect Squares

Recall:

To estimate a perfect square, we locate the perfect square just above and just below the number you are looking for.



For example.

$\sqrt{25} = 5$

a) $\sqrt{17}$ ≈ 4.2

$\sqrt{16} = 4$

8 $\sqrt{36} = 6$

b) $\sqrt{44}$ ≈ 6.7

5 $\sqrt{49} = 7$

4 $\sqrt{100} = 10$

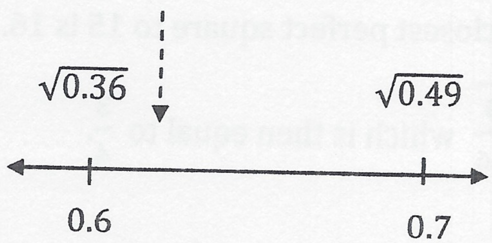
c) $\sqrt{104}$ ≈ 10.15 or 10.2

17 $\sqrt{121} = 11$

We can use the same way we estimate non-perfect square whole numbers, to estimate the square root of non-perfect fractions and decimals.

Consider $\sqrt{0.39}$.

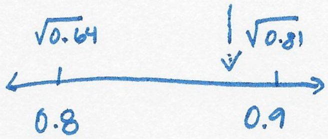
Between which two decimal perfect squares does 0.38 fall?



Example 1:

Estimate each square root and show workings.

a) $\sqrt{0.79}$



- $\sqrt{0.79}$ is between $\sqrt{0.64}$ and $\sqrt{0.81}$
- $\sqrt{0.79}$ is between 0.8 and 0.9

$\therefore \approx 0.88$

b) $\sqrt{0.5}$

- $\sqrt{0.5}$ is between $\sqrt{0.49}$ and $\sqrt{0.64}$
- $\sqrt{0.5}$ is between 0.7 and 0.8

≈ 0.71

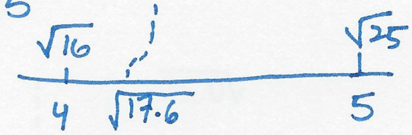
c) $\sqrt{7.8}$

- 7.8 is between 4 and 9

so $\sqrt{7.8} \approx 2.8$

b) $\sqrt{17.6}$

- 17.6 is between 16 and 25



- so $\sqrt{17.6} \approx 4.2$

To estimate a fraction, we find the closest perfect squares to the numerator and the denominator and find the square root.

Consider $\sqrt{\frac{8}{15}}$.

The closest perfect square to 8 is 9. The closest perfect square to 15 is 16.

Therefore, we can estimate $\sqrt{\frac{8}{15}}$ as $\sqrt{\frac{9}{16}}$ which is then equal to $\frac{3}{4}$.

Try the following:

a) $\sqrt{\frac{15}{24}} \approx \sqrt{\frac{16}{25}}$

then

$\sqrt{\frac{15}{24}} \approx \sqrt{\frac{16}{25}} = \frac{4}{5}$

b) $\sqrt{\frac{54}{79}} \approx \sqrt{\frac{49}{81}}$

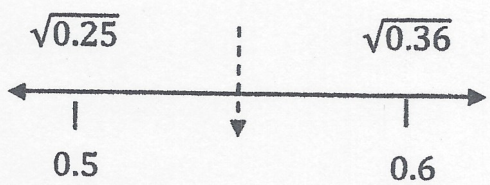
then

$\sqrt{\frac{49}{81}} \approx \sqrt{\frac{54}{79}} = \frac{7}{9}$

We can also estimate fractions by changing the fraction to a decimal first, and then estimating.

Consider $\sqrt{\frac{3}{10}}$. $\frac{3}{10}$ is the same as 0.3.

Since 0.30 falls between the perfect squares 0.25 and 0.36, $\sqrt{0.30}$ falls between $\sqrt{0.25}$ and $\sqrt{0.36}$ so we estimate in between these numbers.



Using a method of your choice, estimate each of the following:

Find each square root:

a) $\sqrt{\frac{8}{79}} \approx \sqrt{\frac{9}{81}}$

then

$\frac{3}{9} \approx \frac{1}{3}$

b) $\sqrt{\frac{5}{12}} \approx \sqrt{\frac{9}{16}} \approx \frac{3}{4}$

$\frac{5}{12}$ is equivalent to

$\frac{5}{12} \times \frac{5}{5} = \sqrt{\frac{25}{60}}$, or ≈ 0.75

c) $\sqrt{\frac{13}{4}} = \sqrt{3.25}$

$3.25 = \frac{325}{100}$

So

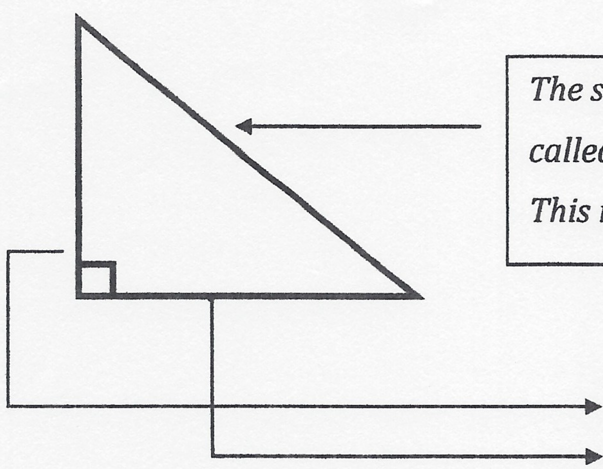
$\sqrt{\frac{13}{4}} = \sqrt{\frac{325}{100}}$

$\sqrt{3.25}$ is between $\sqrt{1} = 1$ and $\sqrt{4} = 2$

so $\sqrt{3.25} \approx 1.80$

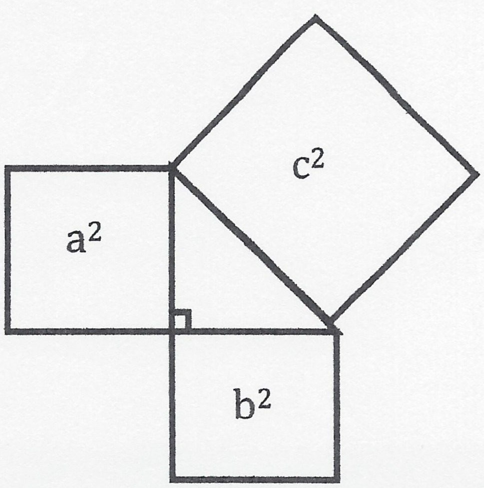
Recall: Pythagorean Theorem

A right triangle is a triangle with one 90° angle. The Pythagorean Theorem states that for any right triangle, the area of the square on the hypotenuse is equal to the sum of the area of the squares on the other two sides (legs).



The side directly across from the 90° angle is called the HYPOTENUSE.
This is always the longest side in a right triangle.

The two remaining sides are called LEGS.



$$a^2 + b^2 = c^2$$

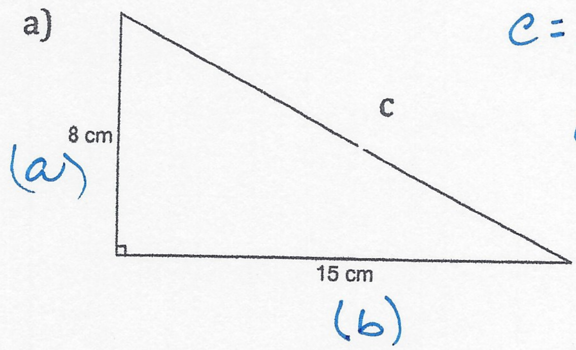
Example 1:

Where
c= hypotenuse

a) Find the missing value:

$$c^2 = a^2 + b^2$$

a)



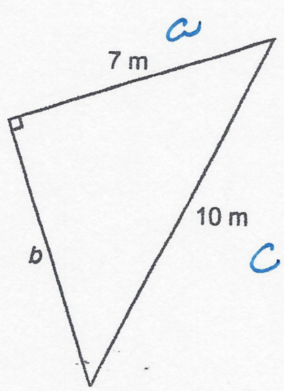
$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{8^2 + 15^2}$$

$$c = \sqrt{64 + 225} = \sqrt{289\ cm^2}$$

$$c = 17\ cm$$

b)



$$b^2 = c^2 - a^2$$

$$b = \sqrt{(10)^2 - (7)^2}$$

$$= \sqrt{100 - 49} = \sqrt{51}$$

$$b = 7.14\ cm$$