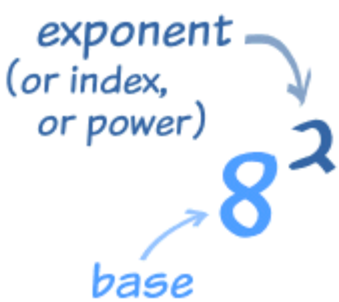
Laws of Exponents

Exponents are also called **Powers** or **Indices**



The exponent of a number says **how many times** to use the number in a **multiplication.**

In this example: $\mathbf{8^2 = 8 \times 8 = 64}$

- In words: 8^2 could be called "8 to the second power", "8 to the power 2" or simply "8 squared"

So an Exponent just saves you writing out lots of multiplies!

Example: a^7

$$a^7 = a \times a \times a \times a \times a \times a \times a = aaaaaaa$$

Notice how I just wrote the letters together to mean multiply? We will do that a lot here.

Example: $x^6 = xxxxxx$

The Key to the Laws

Writing all the letters down is the key to understanding the Laws

Example: $x^2x^3 = (xx)(xxx) = xxxxx = x^5$

So, when in doubt, just remember to write down all the letters (as many as the exponent tells you to) and see if you can make sense of it.

All you need to know ...

The "Laws of Exponents" (also called "Rules of Exponents") come from three ideas:

The exponent says **how many times** to use the number in a multiplication**.**

 A **negative exponent** means **divide**, because the opposite of multiplying is dividing

 A fractional exponent like **1/n** means to **take the nth root**: $x^{\frac{1}{n}} = \sqrt[n]{x}$

If you understand those, then you understand exponents!

And all the laws below are based on those ideas.

Laws of Exponents

Here are the Laws (explanations follow):

Law	**Example**
$x^1 = x$	$6^1 = 6$
$x^0 = 1$	$7^0 = 1$
$x^m x^n = x^{m+n}$	$x^2 x^3 = x^{2+3} = x^5$
$x^m/x^n = x^{m-n}$	$x^6/x^2 = x^{6-2} = x^4$
$(x^m)^n = x^{mn}$	$(x^2)^3 = x^{2\times 3} = x^6$
$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
$(x/y)^n = x^n/y^n$	$(x/y)^2 = x^2 / y^2$

Laws Explained

The first three laws above ($x^1 = x$, $x^0 = 1$ and $x^{-1} = 1/x$) are just part of the natural sequence of exponents. Have a look at this example:

Example: Powers of 5

	.. etc..	
5^2	**1 × 5 × 5**	25
5^1	**1 × 5**	5
5^0	**1**	1
5^{-1}	**1 ÷ 5**	0.2
5^{-2}	**1 ÷ 5 ÷ 5**	0.04
	.. etc..	

You will see that positive, zero or negative exponents are really part of the same pattern, i.e. 5 times larger (or smaller) depending on whether the exponent gets larger (or smaller).

The law that $x^m x^n = x^{m+n}$

With $x^m x^n$, how many times will you end up multiplying "x"? *Answer:* first "m" times, then **by another** "n" times, for a total of "m+n" times.

Example: $x^2x^3 = (xx)(xxx) = xxxxx = x^5$

So, $x^2x^3 = x^{(2+3)} = x^5$

The law that $x^m/x^n = x^{m-n}$

Like the previous example, how many times will you end up multiplying "x"? Answer: "m" times, then **reduce that** by "n" times (because you are dividing), for a total of "m-n" times.

Example: $x^4/x^2 = (xxxx) / (xx) = xx = x^2 = x^{4-2}$

(Remember that **x**/**x** = 1, so every time you see an **x** "above the line" and one "below the line" you can cancel them out.)

This law can also show you why $\mathbf{x^0=1}$:

Example: $x^2/x^2 = x^{2-2} = x^0 = 1$

The law that $(x^m)^n = x^{mn}$

First you multiply x "m" times. Then you have **to do that "n" times**, for a total of m×n times.

Example: $(x^3)^4 = (xxx)^4 = (xxx)(xxx)(xxx)(xxx) = xxxxxxxxxxxx = x^{12}$

So $(x^3)^4 = x^{3\times4} = x^{12}$

The law that $(xy)^n = x^ny^n$

To show how this one works, just think of re-arranging all the "x"s and "y" as in this example:

Example: $(xy)^3 = (xy)(xy)(xy) = xyxyxy = xxxyyy = (xxx)(yyy) = x^3y^3$

The law that $(x/y)^n = x^n/y^n$

Similar to the previous example, just re-arrange the "x"s and "y"s

Example: $(x/y)^3 = (x/y)(x/y)(x/y) = (xxx)/(yyy) = x^3/y^3$

And That Is It!

If you find it hard to remember all these rules, then remember this:

you can work them out when you understand the
three ideas at the top of this page

Oh, One More Thing ... What if x= 0?

Positive Exponent (n>0)	$0^n = 0$
Negative Exponent (n<0)	**Undefined!** (Because dividing by 0)
Exponent = 0	*Ummm* ... see below!

The Strange Case of 0^0

There are two different arguments for the correct value of 0^0.

0^0 could be 1, or possibly 0, so some people say it is really "indeterminate":

$x^0 = 1$, so ...	$0^0 = 1$
$0^n = 0$, so ...	$0^0 = 0$
When in doubt ...	0^0 = *"indeterminate"*

http://www.mathsisfun.com/algebra/exponent-laws.html