**Laws of Exponents**

Exponents are also called **Powers** or **Indices**

|  |  |
| --- | --- |
| 8 to the Power 2 | The exponent of a number says **how many times** to use the number in a **multiplication.**  In this example: **82 = 8 × 8 = 64**   * In words: 82 could be called "8 to the second power", "8 to the power 2" or simply "8 squared" |

So an Exponent just saves you writing out lots of multiplies!

**Example: a7**

a**7** = a × a × a × a × a × a × a = aaaaaaa

Notice how I just wrote the letters together to mean multiply? We will do that a lot here.

**Example: x6 = xxxxxx**

**The Key to the Laws**

Writing all the letters down is the key to understanding the Laws

**Example: x2x3 = (xx)(xxx) = xxxxx = x5**

So, when in doubt, just remember to write down all the letters (as many as the exponent tells you to) and see if you can make sense of it.

**All you need to know ...**

The "Laws of Exponents" (also called "Rules of Exponents") come from three ideas:

|  |  |
| --- | --- |
| http://www.mathsisfun.com/images/style/pencil-paper.gif | The exponent says **how many times** to use the number in a multiplication**.** |
|  |  |
| http://www.mathsisfun.com/images/style/turn-over.gif | A **negative exponent** means **divide**, because the opposite of multiplying is dividing |
|  |  |
| http://www.mathsisfun.com/images/style/pie-slice.gif | |  |  | | --- | --- | | A [fractional exponent](http://www.mathsisfun.com/algebra/exponent-fractional.html) like **1/n** means to **take the nth root**: | http://www.mathsisfun.com/algebra/images/fractional-exponent.png | |

If you understand those, then you understand exponents!

And all the laws below are based on those ideas.

**Laws of Exponents**

Here are the Laws (explanations follow):

|  |  |
| --- | --- |
| **Law** | **Example** |
| x1 = x | 61 = 6 |
| x0 = 1 | 70 = 1 |
| x-1 = 1/x | 4-1 = 1/4 |
|  |  |
| xmxn = xm+n | x2x3 = x2+3 = x5 |
| xm/xn = xm-n | x6/x2 = x6-2 = x4 |
| (xm)n = xmn | (x2)3 = x2×3 = x6 |
| (xy)n = xnyn | (xy)3 = x3y3 |
| (x/y)n = xn/yn | (x/y)2 = x2 / y2 |
| x-n = 1/xn | x-3 = 1/x3 |
| And the law about Fractional Exponents: | |
| http://www.mathsisfun.com/algebra/images/nth-root-xm.gif | http://www.mathsisfun.com/algebra/images/3rd-root-x2.gif |

**Laws Explained**

The first three laws above (x1 = x, x0 = 1 and x-1 = 1/x) are just part of the natural sequence of exponents. Have a look at this example:

|  |  |  |  |
| --- | --- | --- | --- |
| **Example: Powers of 5** | | | |
|  | .. etc.. |  | http://www.mathsisfun.com/algebra/images/exponent-larger-smaller.gif |
| **52** | **1 × 5 × 5** | 25 |
| **51** | **1 × 5** | 5 |
| **50** | **1** | 1 |
| **5-1** | **1 ÷ 5** | 0.2 |
| **5-2** | **1 ÷ 5 ÷ 5** | 0.04 |
|  | .. etc.. |  |

You will see that positive, zero or negative exponents are really part of the same pattern, i.e. 5 times larger (or smaller) depending on whether the exponent gets larger (or smaller).

**The law that xmxn = xm+n**

With xmxn, how many times will you end up multiplying "x"? *Answer:* first "m" times, then **by another** "n" times, for a total of "m+n" times.

**Example: x2x3 = (xx)(xxx) = xxxxx = x5**

So, x2x3 = x(2+3) = x5

**The law that xm/xn = xm-n**

Like the previous example, how many times will you end up multiplying "x"? Answer: "m" times, then **reduce that** by "n" times (because you are dividing), for a total of "m-n" times.

**Example: x4/x2 = (xxxx) / (xx) = xx = x2 = x4-2**

(Remember that **x**/**x** = 1, so every time you see an **x** "above the line" and one "below the line" you can cancel them out.)

This law can also show you why **x0=1** :

**Example: x2/x2 = x2-2 = x0 =1**

**The law that (xm)n = xmn**

First you multiply x "m" times. Then you have **to do that "n" times**, for a total of m×n times.

**Example: (x3)4 = (xxx)4 = (xxx)(xxx)(xxx)(xxx) = xxxxxxxxxxxx = x12**

So (x3)4 = x3×4 = x12

**The law that (xy)n = xnyn**

To show how this one works, just think of re-arranging all the "x"s and "y" as in this example:

**Example: (xy)3 = (xy)(xy)(xy) = xyxyxy = xxxyyy = (xxx)(yyy) = x3y3**

**The law that (x/y)n = xn/yn**

Similar to the previous example, just re-arrange the "x"s and "y"s

**Example: (x/y)3 = (x/y)(x/y)(x/y) = (xxx)/(yyy) = x3/y3**

**The law that fractional exponents**

OK, this one is a little more complicated!

I suggest you read [Fractional Exponents](http://www.mathsisfun.com/algebra/exponent-fractional.html) first, or this may not make sense.

Anyway, the important idea is that:

x1/**n** = The **n-**th Root of x

And so a fractional exponent like 43/2 is really saying to do a **cube** (3) and a **square root** (1/2), in any order.

Just remember from fractions that **m/n = m × (1/n)**:

**Example: fractional exponents**

The order does not matter, so it also works for **m/n = (1/n) × m**:

**Example: fractional exponents**

**And That Is It!**

*If you find it hard to remember all these rules, then remember this:*

you can work them out when you understand the   
three ideas at the top of this page

**Oh, One More Thing ... What if x= 0?**

|  |  |
| --- | --- |
| Positive Exponent (n>0) | 0n = 0 |
| Negative Exponent (n<0) | **Undefined!** (Because dividing by 0) |
| Exponent = 0 | *Ummm ...* see below! |

**The Strange Case of 00**

There are two different arguments for the correct value of 00.

00 could be 1, or possibly 0, so some people say it is really "indeterminate":

|  |  |  |
| --- | --- | --- |
| http://www.mathsisfun.com/images/style/question.gif | x0 = 1, so ... | 00 = 1 |
| 0n = 0, so ... | 00 = 0 |
| When in doubt ... | 00 = *"indeterminate"* |

<http://www.mathsisfun.com/algebra/exponent-laws.html>

|  |  |  |  |
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| http://www.math-play.com/mm_spacer.gif | http://www.math-play.com/mm_spacer.gif   |  | | --- | | **EXPONENT GAME** | | Play this fun exponent game with Kiwi to practice the six rules of exponents.    Do you know the six rules of exponents? Here they are:  http://www.math-play.com/image-exponents-rules.jpg  http://www.math-play.com/exponent-game.html | |