

## Grade 9 Math

### Unit 1: Square Roots and Surface Area.

#### Review from Grade 8: Perfect Squares

What is a perfect square?

- ▶ Perfect square numbers are formed when we multiply a number (factor) by itself, or square a number.

For Example:  $3 \times 3 = 9$

9 is a perfect square, and 3 is it's factor.

There are other ways to ask the same question....

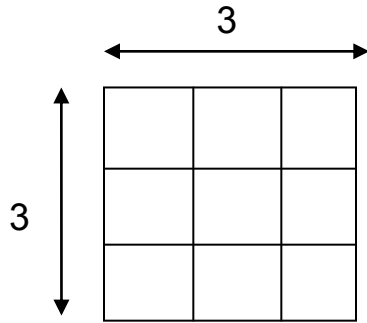
- ▶ What is the square of 3?      Meaning what is  $3 \times 3$ , or what is  $3^2 = 9$
- ▶ What is 3 squared ?      Meaning what is  $3 \times 3$ , or what is  $3^2 = 9$
- ▶ What is 3 to the power of 2? Meaning what is  $3 \times 3$ , or what is  $3^2 = 9$

We can sketch a diagram of perfect squares, by actually drawing squares. The factors ( the number that multiplies by itself ) are the side length of the square and the area of the square is the perfect square number.

Length  $\times$  Length = Area of a Square

$$(\text{Length})^2 = \text{Area}$$

$$\begin{array}{ccc} 3 & \times & 3 & = & 9 \\ \uparrow & & \uparrow & & \uparrow \\ \text{Length} & & \text{Length} & & \text{Area} \end{array}$$

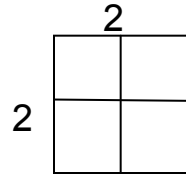


and there are 9 little squares

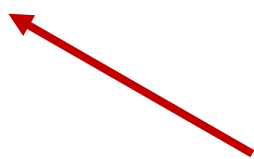
$$3 \times 3 = 9$$

The List of Perfect Squares from 1 to 20.

- $1^2 = 1 \times 1 = 1$
- $2^2 = 2 \times 2 = 4$
- $3^2 = 3 \times 3 = 9$
- $4^2 = 4 \times 4 = 16$
- $5^2 = 5 \times 5 = 25$
- $6^2 = 6 \times 6 = 36$
- $7^2 = 7 \times 7 = 49$
- $8^2 = 8 \times 8 = 64$
- $9^2 = 9 \times 9 = 81$
- $10^2 = 10 \times 10 = 100$
- $11^2 = 11 \times 11 = 121$
- $12^2 = 12 \times 12 = 144$
- $13^2 = 13 \times 13 = 169$
- $14^2 = 14 \times 14 = 196$
- $15^2 = 15 \times 15 = 225$
- $16^2 = 16 \times 16 = 256$
- $17^2 = 17 \times 17 = 289$
- $18^2 = 18 \times 18 = 324$
- $19^2 = 19 \times 19 = 361$
- $20^2 = 20 \times 20 = 400$



... etc



These are the perfect square numbers.

## Review from Grade 8: Square Root

When we multiply a number by itself we find the perfect square

$$10^2 = 10 \times 10 = 100$$

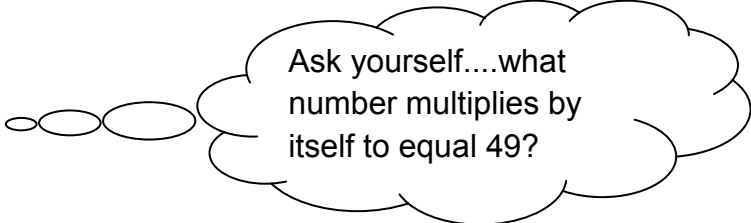
Finding the square root of a number is doing the opposite. We are given the perfect square and asked to find what number multiplied by itself to get that number.

Finding the perfect square and finding the square root are called **inverse operations**. ( they are opposites ).

The symbol for square root is  $\sqrt{\quad}$

$$\sqrt{100} = \sqrt{10 \times 10} = 10$$

What is  $\sqrt{49}$  ?



Ask yourself....what number multiplies by itself to equal 49?

## Sec 1.1: Square Roots of Perfect Squares.

### Review from Grade 8: Decimals and Fractions

How to change a decimal to a fraction:

A). 0.6



The 6 is in the first decimal place called the **tenths** place. Therefore,

$$0.6 = \frac{6}{10}$$

B). 0.08



The 8 is in the second decimal place called the **hundredths** place.

Therefore,

$$0.08 = \frac{8}{100}$$

C). 0.25



The 5 is in the hundredths place, therefore,

$$0.25 = \frac{25}{100}$$

Always look at the last number and that's the decimal position we are looking for

D). 0.379



The 9 is in the third decimal place, called the thousandths place,

Therefore,

$$0.379 = \frac{379}{1000}$$

#### Remember:

$$0.1 = \frac{1}{10} \quad (\text{tenth})$$

$$0.01 = \frac{1}{100} \quad (\text{hundredth})$$

$$0.001 = \frac{1}{1000} \quad (\text{thousandth})$$

Some fractions and decimals can also be perfect squares. If we can represent the area using squares than it is a perfect square.

To determine if a fraction is a perfect square, we need to find out if the numerator (top number) and the denominator (bottom number) are both perfect squares.

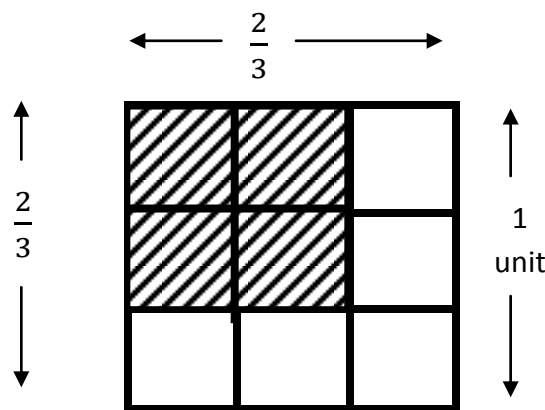
Examples of Fractions:

1. Is  $\frac{4}{9}$  a perfect square?

- Since  $\sqrt{4} = 2$  and  $\sqrt{9} = 3$  then  $\frac{4}{9}$  is a perfect square

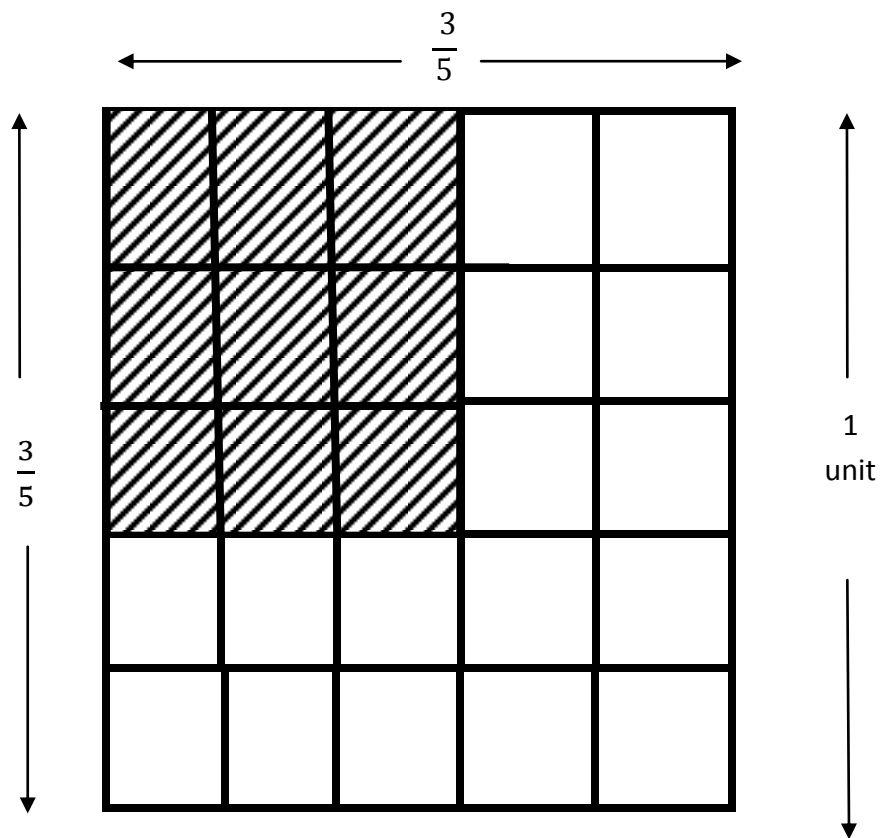
$$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3} \quad \text{Check your answer} \quad \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

This can also be represented by drawing a diagram using squares:



There are 2 out of 3 squares shaded along the width and length of the square and there are 4 squares shaded out of a total of 9 squares. And it still created a square.

2. Use a diagram to determine the value of  $\sqrt{\frac{9}{25}}$  ?



$$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$$

3. Is  $16\frac{4}{9}$  a perfect square?

FIRST we must change this mixed number to an improper fraction.

$$16\frac{4}{9} = \frac{148}{9}$$

Are both the numerator (148) and denominator (9) perfect squares?

No! 148 is not a perfect square therefore,  $16\frac{4}{9}$  is not either.

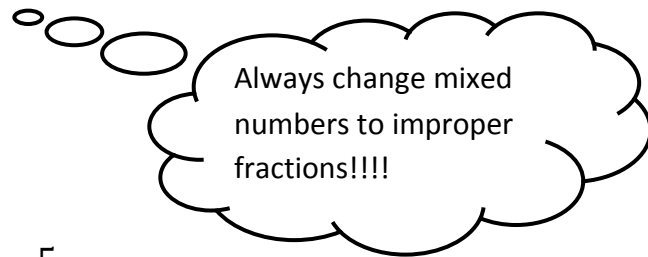
**\*\*\*NOTE\*\*\*** Just because 16, 4 and 9 are individually perfect squares, it did not necessarily mean that  $16\frac{4}{9}$  is automatically a perfect square too. **YOU MUST CHANGE TO IMPROPER FRACTION** to get the correct answer.

4a. Is  $4\frac{21}{25}$  a perfect square?

$$4\frac{21}{25} = \frac{121}{25}$$

$$\sqrt{121} = 11 \quad \text{and} \quad \sqrt{25} = 5$$

Therefore,  $\sqrt{\frac{121}{25}} = \frac{\sqrt{121}}{\sqrt{25}} = \frac{11}{5}$  It is a perfect square.



4b. Is  $\frac{8}{50}$  a perfect square?

$\sqrt{\frac{8}{50}}$  this doesn't work however, if you reduce the fractions to lowest terms

$\sqrt{\frac{8}{50}} = \sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$  so it actually is a perfect square. BE CAREFUL!!!!

Examples of Decimals:

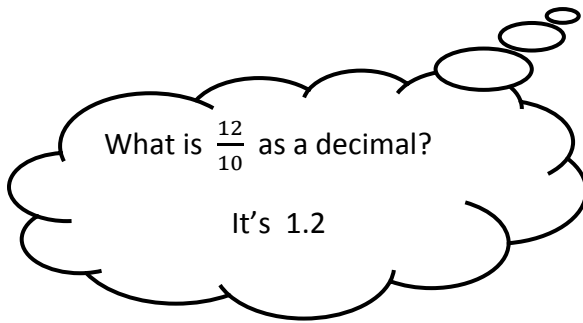
5. Find  $\sqrt{1.44}$

- There are a couple of ways to approach this question.

First change 1.44 to a fraction.  $1.44 = \frac{144}{100}$

Then determine if the numerator and denominator are perfect squares.

$$\sqrt{\frac{144}{100}} = \frac{\sqrt{144}}{\sqrt{100}} = \frac{12}{10} \quad \text{Therefore, it is a perfect square.}$$



- Another way to complete this question is to recognize that  $12 \times 12 = 144$  and that  $1.2 \times 1.2 = 1.44$ , so 1.44 is a perfect square.

6. Which decimal is a perfect square 6.4 or 0.64? Justify your answer.

$$6.4 = \frac{64}{10} \quad \sqrt{\frac{64}{10}} = \frac{\sqrt{64}}{\sqrt{10}} = \frac{8}{\sqrt{10}}$$

since 10 is not a perfect square than 6.4 is not a perfect square.

$$0.64 = \frac{64}{100} \quad \sqrt{\frac{64}{100}} = \frac{\sqrt{64}}{\sqrt{100}} = \frac{8}{10}$$

Therefore, 0.64 is a perfect square.



Examples of square roots and perfect squares.

$$1. \sqrt{8100} = 90$$

$$\sqrt{81} = 9$$

$$\sqrt{0.81} = 0.9$$

$$\sqrt{0.0081} = 0.09$$

\*\* Many students find it tricky....where does the decimal go?

Here's a hint ... if the perfect square is a whole number, than the square root answer is smaller than the original number.

$$\sqrt{81} = 9 \quad (9 \text{ is less than } 81)$$

... if the perfect square is a rational number (decimal or fraction) between 0 and 1, than the square root is bigger than the original number.

$$\sqrt{0.81} = 0.9 \quad (0.9 \text{ is greater than } 0.81)$$

When finding a square root, you find the number that multiplies by itself.

$$\sqrt{81} = 9 \quad \text{because} \quad 9 \times 9 = 81$$

What about  $-9$  ?

$$\text{Can } \sqrt{81} = -9 \quad \text{because} \quad -9 \times -9 = 81 \text{ ?}$$

YES! Square roots can have negative answers, but for us we will only be finding the principal square root and that's the positive answer.

2. Calculate the number whose square root is:

a).  $\frac{17}{5}$

b). 1.21

c). 0.5

$$\frac{17}{5} \times \frac{17}{5} = \frac{289}{25}$$

$$\begin{array}{r} 1.21 \\ \times 1.21 \\ \hline 1.4641 \end{array}$$

$$\begin{array}{r} 0.5 \\ \times 0.5 \\ \hline 0.25 \end{array}$$

Just multiply each number by itself.

The List of Some Perfect Squares Decimal Numbers.

$0.1^2$	=	$0.1 \times 0.1$	=	0.01
$0.2^2$	=	$0.2 \times 0.2$	=	0.04
$0.3^2$	=	$0.3 \times 0.3$	=	0.09
$0.4^2$	=	$0.4 \times 0.4$	=	0.16
$0.5^2$	=	$0.5 \times 0.5$	=	0.25
$0.6^2$	=	$0.6 \times 0.6$	=	0.36
$0.7^2$	=	$0.7 \times 0.7$	=	0.49
$0.8^2$	=	$0.8 \times 0.8$	=	0.64
$0.9^2$	=	$0.9 \times 0.9$	=	0.81
$1.1^2$	=	$1.1 \times 1.1$	=	1.21
$1.2^2$	=	$1.2 \times 1.2$	=	1.44
$1.3^2$	=	$1.3 \times 1.3$	=	1.69
$1.4^2$	=	$1.4 \times 1.4$	=	1.96
$1.5^2$	=	$1.5 \times 1.5$	=	2.25
$1.6^2$	=	$1.6 \times 1.6$	=	2.56
$1.7^2$	=	$1.7 \times 1.7$	=	2.89
$1.8^2$	=	$1.8 \times 1.8$	=	3.24
$1.9^2$	=	$1.9 \times 1.9$	=	3.61

These are the perfect square numbers.

3. Determine whether each decimal is a perfect square.

*You can use a calculator to find out if a decimal is a perfect square.*

The square root of a **perfect square** decimal is either a

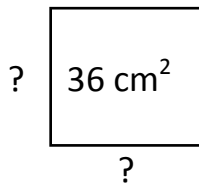
- **terminating decimal** (ends after a certain number of decimal places) or
- a **repeating decimal** (has a repeating pattern of digits in the decimal).

Decimal	Value of Square Root	Type of Decimal	Is decimal a perfect square?
1.69	1.3	Terminating	Yes
3.5	1.8708286...	Non-terminating Non-repeating	No
70.5	8.3964278...	Non-terminating Non-repeating	No
5.76	2.4	Terminating	Yes
0.25	0.5	Terminating	Yes
2.5	1.5811388...	Non-terminating Non-repeating	No

## Sec 1.2: Square Roots of Non - Perfect Squares.

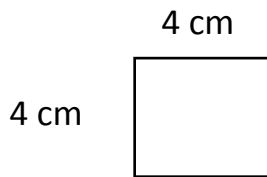
A non-perfect square is a number that cannot be written as a product of two equal numbers.

1. If the area of a square is  $36 \text{ cm}^2$ , what is the side length of the square?



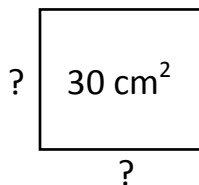
$$\begin{aligned}\text{side length} &= \sqrt{36} \\ &= 6 \text{ cm}\end{aligned}$$

2. If the side length of a square is 4 cm, what is the area of the square?



$$\begin{aligned}\text{Area} &= \text{length} \times \text{length} \\ &= 4 \times 4 \\ &= 16 \text{ cm}^2\end{aligned}$$

3. If the area of a square is  $30 \text{ cm}^2$ , what is the side length of the square?



$$\begin{aligned}\text{side length} &= \sqrt{30} \\ &= ???\end{aligned}$$

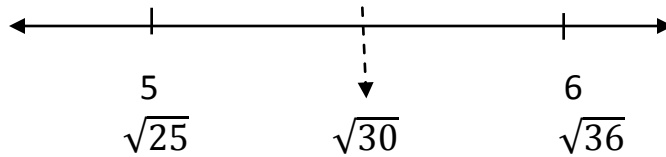
This is not a perfect square so we can't get an exact answer. But we can estimate using the perfect squares we do know.

### Perfect Squares

1  
4  
9  
16  
25  
36  
49  
64  
81  
100  
... etc

Between which two consecutive perfect squares does 30 fall ?

Imagine a number line and use the knowledge that  $\sqrt{25} = 5$  and  $\sqrt{36} = 6$ . Therefore, the  $\sqrt{30}$  must lie between 5 and 6.



$\sqrt{30}$  falls approximately half way between 5 and 6.  $5 < \sqrt{30} < 6$

So let's make a guess and check our answer.

$$\begin{array}{r} 5.5 \\ \times 5.5 \\ \hline 30.25 \end{array}$$

$$\begin{array}{r} 5.4 \\ \times 5.4 \\ \hline 29.16 \end{array}$$

$$\begin{array}{r} 5.6 \\ \times 5.6 \\ \hline 31.36 \end{array}$$

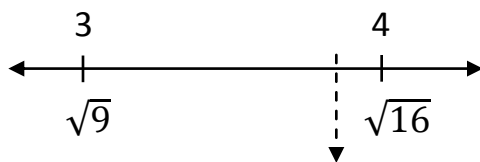
The closest estimate is 5.5, therefore  $\sqrt{30} \sim 5.5$

↑  
This symbol means approximately. ( $\approx$  or  $\sim$ )

4. What is  $\sqrt{15}$  ?

- Since 15 is not a perfect square we must estimate. Between what two perfect squares does 15 fall between?

15 falls between 9 and 16, so  $\sqrt{15}$  falls between  $\sqrt{9}$  and  $\sqrt{16}$  or 3 and 4.  $3 < \sqrt{15} < 4$



$\sqrt{15}$  is really close to 4 (which is  $\sqrt{16}$ ). Let's make a guess and check.

$$\begin{array}{r} 3.8 \\ \times 3.8 \\ \hline 14.44 \end{array}$$

$$\begin{array}{r} 3.9 \\ \times 3.9 \\ \hline 15.21 \end{array}$$

$$\begin{array}{r} 3.85 \\ \times 3.85 \\ \hline 14.8225 \end{array}$$

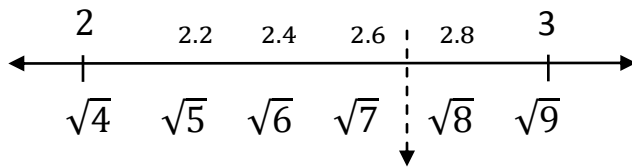
$$\begin{array}{r} 3.87 \\ \times 3.87 \\ \hline 14.9769 \end{array}$$

3.9 is a good estimate but you can take it further if you want.

5. What is  $\sqrt{7.5}$  ?

- Since 7.5 is not a perfect square we must estimate. Between what two perfect squares does 7.5 fall between?

7.5 falls between 4 and 9, so  $\sqrt{7.5}$  falls between  $\sqrt{4}$  and  $\sqrt{9}$   
or 2 and 3.  $2 < \sqrt{7.5} < 3$



$\sqrt{7.5}$  falls between 2.6 and 2.8, so let's make a guess and check.

Try:

$$\begin{array}{r} 2.7 \\ \times 2.7 \\ \hline 7.29 \end{array}$$

use your calculator to check

$$\sqrt{7.5} = 2.7386127 \dots \text{etc}$$

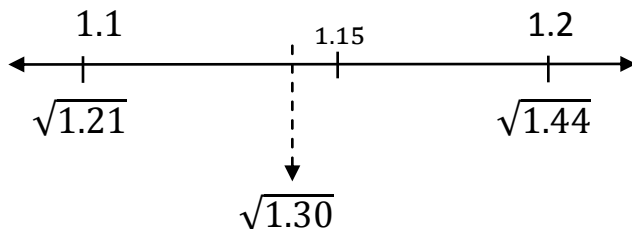
So 2.7 is a really good guess.

6. What is  $\sqrt{1.30}$  ?

Refer to your list of decimal perfect squares.

If you find it hard remembering the decimal perfect square list, [divide all your perfect squares by 100 \(move the decimal 2 places to the left\)](#) you will then have a list of [Decimal Perfect Squares](#)

1.30 falls between 1.21 and 1.44 ..... meaning  $\sqrt{1.21} < \sqrt{1.30} < \sqrt{1.44}$



Looks like  $\sqrt{1.30}$  is approximately 1.14, but let's check our answer!

$$\begin{array}{r} 1.14 \\ \times 1.14 \\ \hline 1.2996 \end{array}$$

Use your calculator to check

$$\sqrt{1.30} = 1.1401754 \dots$$

1.14 is a good guess

## Estimating square roots of non-perfect square fractions

1. What is  $\sqrt{\frac{14}{22}}$  ?

► to estimate this question we can identify the perfect squares closest to 14 and 22 , which are 16 and 25.

► Now let's find  $\sqrt{\frac{16}{25}} = \frac{4}{5}$

► Therefore,  $\sqrt{\frac{14}{22}}$  is approximately  $\frac{4}{5}$

2. Estimate each square root.

A)  $\sqrt{\frac{8}{15}}$

$$\sim \sqrt{\frac{9}{16}} = \frac{3}{4}$$

B).  $\sqrt{\frac{10}{50}}$

$$\sim \sqrt{\frac{9}{49}} = \frac{3}{7}$$

C).  $\sqrt{\frac{30}{12}}$

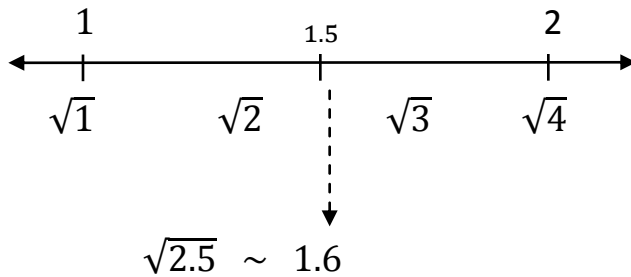
► this fraction is harder to estimate using perfect square benchmarks because 30 is close to 25 and 36 so which number should we chose and 12 is close to 9 and 16, so which number should we chose.

► when this happens there is another way we can approach this question. We can change the fraction to a decimal and estimate the square root of the decimal number instead.

$$\frac{30}{12} = 12\sqrt{\frac{2.5}{30.0}}$$

$$\begin{array}{r} -24 \\ 60 \\ -60 \\ \hline 0 \end{array}$$

►  $\sqrt{2.5}$       $1 < 2.5 < 4$



Check:

$$\begin{array}{r} 1.6 \\ \times 1.6 \\ \hline 2.56 \end{array}$$

$$\begin{array}{r} 1.5 \\ \times 1.5 \\ \hline 2.25 \end{array}$$

↙ Good estimate

3. Use your calculator to determine each answer. What do you notice?

a).  $\sqrt{0.64}$

d).  $\sqrt{38.44}$

b).  $\sqrt{3.9}$

e).  $\sqrt{74.5}$

c).  $\sqrt{15.4}$

f).  $\sqrt{16.81}$



## ANSWERS

a).  $\sqrt{0.64} = 0.8$

d).  $\sqrt{38.44} = 6.2$

b).  $\sqrt{3.9} = 1.8973665$

e).  $\sqrt{74.5} = 8.6313382$

c).  $\sqrt{15.4} = 3.9242833$

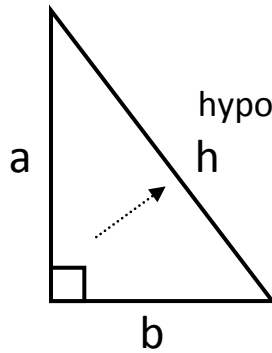
f).  $\sqrt{16.81} = 4.1$

What do you notice?

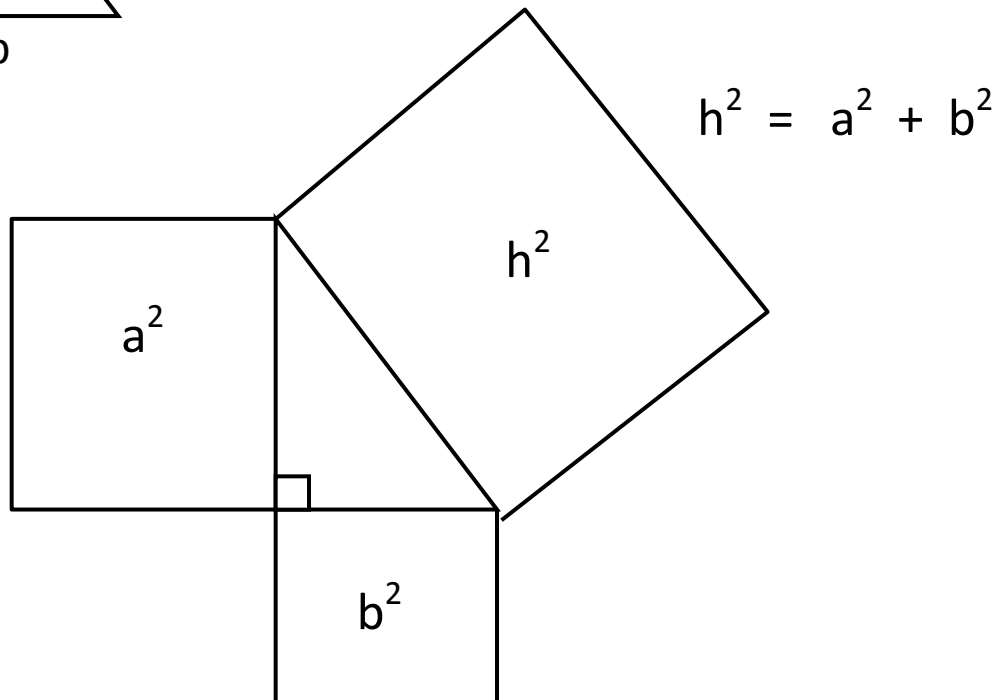
- ▶ When you found the square root and the answer was a **terminating decimal** (a decimal that stopped) then the original number was a perfect square. The answer was exact. These are called **rational numbers**.
- ▶ When you found the square root and the answer was a **non-terminating** (non-stopping) and **non-repeating decimal**, the original number was a non-perfect square. These numbers will not have exact answers, we can only estimate them. (When estimating one or two decimal places are enough). These types of numbers are called **irrational numbers**.

## Review: Pythagorean Theorem

Pythagorean Theorem is a rule which states that, for any right triangle, the area of the square on the hypotenuse is equal to the sum of the area of the squares on the other two sides (legs).

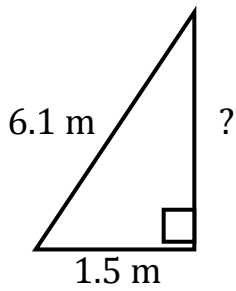


hypotenuse - is always the longest side of a right triangle. It's the side across from the  $90^\circ$  angle.

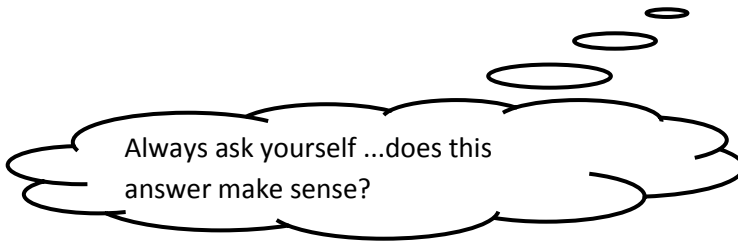


### Examples

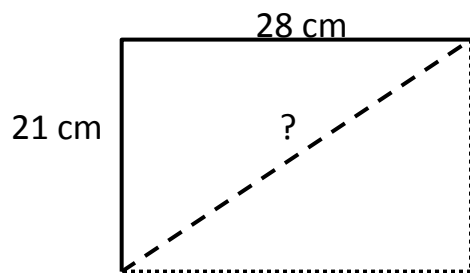
1. A ladder is 6.1 m long. The distance from the base of the ladder to the wall is 1.5 m. How far up the wall will the ladder reach? Hint: sketch a diagram.



$$\begin{aligned}
 h^2 &= a^2 + b^2 \\
 6.1^2 &= 1.5^2 + b^2 \\
 37.21 &= 2.25 + b^2 \\
 37.21 - 2.25 &= b^2 \\
 34.96 &= b^2 \\
 \sqrt{34.96} &= b \\
 b &= 5.9 \text{ m}
 \end{aligned}$$



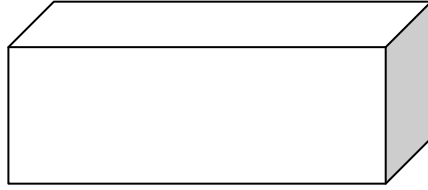
2. The dimensions of a computer monitor are 28 cm by 21 cm. What is the length of the diagonal? Hint: Sketch a diagram.



$$\begin{aligned}
 h^2 &= a^2 + b^2 \\
 h^2 &= 28^2 + 21^2 \\
 h^2 &= 784 + 441 \\
 h^2 &= 1225 \\
 h^2 &= \sqrt{1225} \\
 h &= 35 \text{ cm}
 \end{aligned}$$

### Sec 1.3 Surface Area of Objects Made from Right Rectangular Prisms

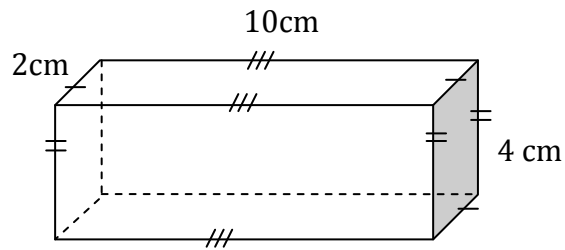
Right Rectangular Prism - a rectangular shaped box. A prism that has rectangular faces.



This prism has 6 faces.

Surface Area – the total area of all the surfaces (faces) of an object.

Ex 1: Find the surface area.



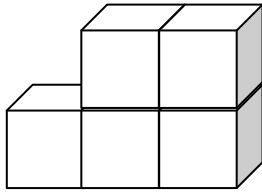
Top	Bottom	Front	Back	Left	Right
$A = L \times W$	$A = L \times W$	$A = L \times W$	$A = L \times W$	$A = L \times W$	$A = L \times W$
$= 2 \times 10$	$= 2 \times 10$	$= 4 \times 10$	$= 4 \times 10$	$= 4 \times 2$	$= 4 \times 2$
$= 20 \text{ cm}^2$	$= 20 \text{ cm}^2$	$= 40 \text{ cm}^2$	$= 40 \text{ cm}^2$	$= 8 \text{ cm}^2$	$= 8 \text{ cm}^2$

$$\text{Total surface Area} = 20 + 20 + 40 + 40 + 8 + 8 = 136 \text{ cm}^2$$

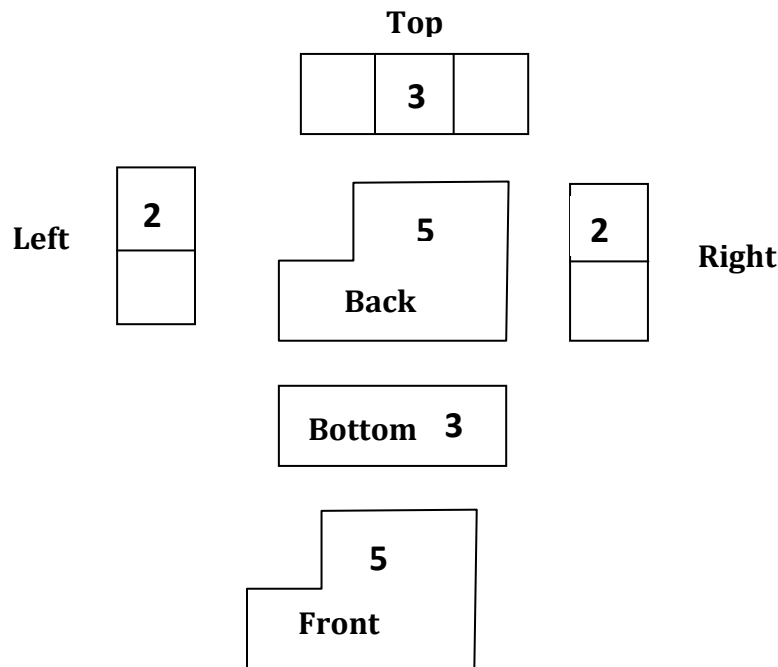
Complete Linking Cubes Activity

There are two methods for finding the area of linking cubes.

**Method #1:** Count the squares on all 6 views of the object.



Top, Bottom, Front, Back, Left, Right



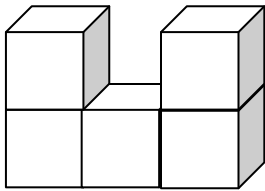
$$\text{Surface Area} = 5 + 5 + 3 + 3 + 2 + 2 = 20 \text{ units}^2$$

**Method #2:** Count the square faces of all the cubes. There are 5 cubes, each with 6 faces, so that's  $5 \times 6 = 30$  faces. Now subtract 2 faces for each place that the squares are joined, or overlap. There are 5 places they are joined, so  $5 \times 2 = 10$  overlapping faces.

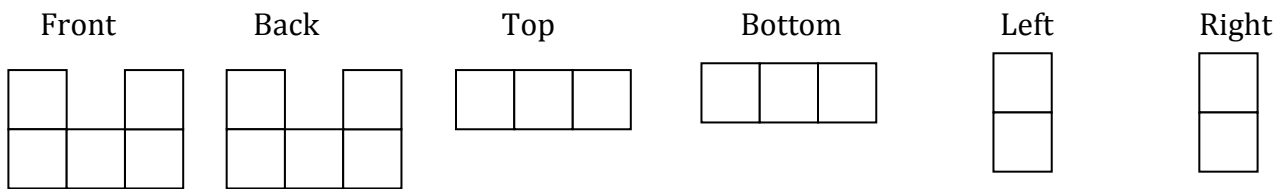
$30 - 10 = 20$  faces in the surface area. So Surface Area is 20 units<sup>2</sup>.

Make sure to complete the following Example if students do not discover it on their own through the Investigation.

Ex: Find the surface area if the side length of each square is 1 cm.



Using Method One: Look at all 6 views



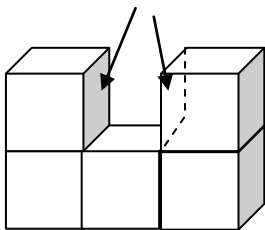
If you count up all the squares you get 20, therefore, the surface area appears to be 20 units<sup>2</sup>.

Try Method Two: Count the square faces of all the cubes. There are 5 cubes, each with 6 faces, so that's  $5 \times 6 = 30$  faces. Now subtract 2 faces for each place that the squares are joined, or overlap. There are 4 places they are joined, so  $4 \times 2 = 8$  overlapping faces.

$30 - 8 = 22$  faces in the surface area. So Surface Area is 22 units<sup>2</sup>.

**QUESTION:** Which answer is correct and why?

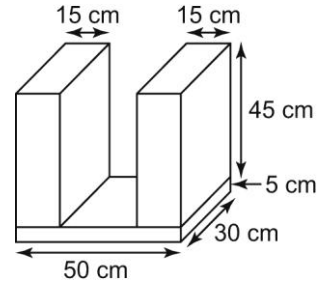
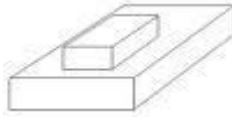
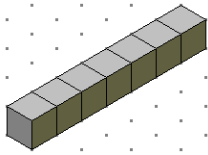
Method two : Surface area of 22 units<sup>2</sup> is correct for this diagram. In method one, by looking at the 6 views of the image, **there are two faces left out**. There are on the inside. To avoid a mistake like this from happening – you should always use method two.



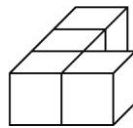
## Determining the Surface Area of Composite Objects

- A **composite object** is an object made up of or composed of more than one object. It may be composed of more than one of the same type of object such as a 'train' of cubes or it could be composed of different types of objects.

Examples:

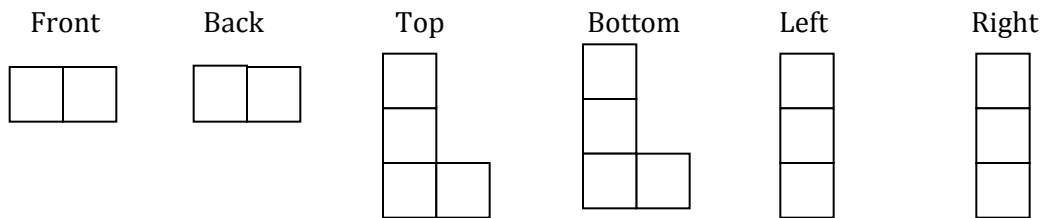


**Example 1.** Determine the surface area of the composite object. Each cube has a length of **2 cm**.



- To find the surface area of the cubes, two strategies can be used.

⇒ **Strategy 1:** Draw the views of the object and count the squares.

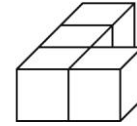


- 18 squares
- Each square has an area of  $2\text{cm} \times 2\text{cm} = 4\text{ cm}^2$
- Total Surface Area =  $18 \times 4\text{ cm}^2 = 72\text{ cm}^2$

⇒ **Strategy 2:** Count the square faces of all the cubes and subtract 2 faces for each surface where the cubes are joined.

There are 4 cubes each with 6 faces.

The total number of faces =  $4 \times 6 = 24$



Question: In how many places do the faces overlap?  
3 places which equals  $3 \times 2 = 6$  faces.

There are 6 faces that **cannot be included** in the surface area. Therefore

$24 - 6 = 18$  faces should be included.

Each face has a surface area of  $2 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$

$18 \text{ faces} \times 4 \text{ cm}^2 = 72 \text{ cm}^2$

**Example 2.** Two rectangular prisms are used to build stairs for a dollhouse.

- (i) Determine the surface area of the stairs.
- (ii) Can the stairs be carpeted with  $200 \text{ cm}^2$  (or  $0.02 \text{ m}^2$ ) of carpet?

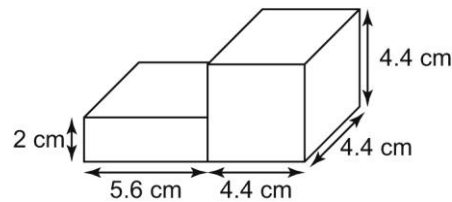
(i) Surface Area of small rectangular prism

Front/back =  $2(5.6 \times 2) = 22.4 \text{ cm}^2$

Top/bottom =  $2(5.6 \times 4.4) = 49.3 \text{ cm}^2$

Sides =  $2(2 \times 4.4) = 17.6 \text{ cm}^2$

Total small prism =  $89.3 \text{ cm}^2$



Surface Area of large Prism

Front/back =  $2(4.4 \times 4.4) = 38.72 \text{ cm}^2$

Top/bottom =  $2(4.4 \times 4.4) = 38.72 \text{ cm}^2$

Sides =  $2(4.4 \times 4.4) = 38.72 \text{ cm}^2$

Total large prism =  $116.2 \text{ cm}^2$

\*Must subtract areas of overlap between two prisms:  $2(2 \text{ cm} \times 4.4 \text{ cm}) = 17.6 \text{ cm}^2$

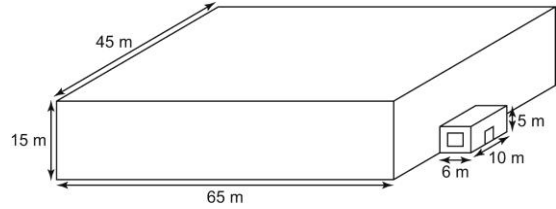
Total Surface area of both prisms =  $89.3 \text{ cm}^2 + 116.2 \text{ cm}^2 - 17.6 \text{ cm}^2 = 187.9 \text{ cm}^2$

- (ii) With  $200 \text{ cm}^2$  (or  $0.02 \text{ m}^2$ ) of carpet there is enough to cover the stairs.  
Or convert  $187.9 \text{ cm}^2$  to  $\text{m}^2 = 0.01879 \text{ m}^2$  Enough to cover stairs!



**Example 3.** The local hockey rink is shown in the diagram at the right. It is to be painted.

- a) Determine the surface area of the structure.
- b) The roof, windows, and door are not to be painted. The door is 1 m by 2 m and the window is 4 m by 2 m. Determine the surface area to be painted.
- c) A can of paint covers  $300 \text{ m}^2$  and costs \$45. Determine the cost of the paint needed.



**Answer:**

- a) The 4 walls and the roof of the rink form the surface area.

Area of roof	= $65\text{m} \times 45\text{m}$	= $2925\text{m}^2$
Area of left and right side walls	= $2(65\text{m} \times 15\text{m})$	= $1950 \text{ m}^2$
Area of front and back walls	= $2(45\text{m} \times 15\text{m})$	= $1350 \text{ m}^2$
<u>Surface area of main area</u>		<b>= <math>6225 \text{ m}^2</math></b>

The 3 walls and the roof of the entrance portion of the rink form its surface area.

Area of roof	= $6\text{m} \times 10\text{m}$	= $60 \text{ m}^2$
Area of front	= $5\text{m} \times 10\text{m}$	= $50 \text{ m}^2$
Area of left and right side walls	= $2(6\text{m} \times 5\text{m})$	= $60 \text{ m}^2$
<u>Surface area of entrance</u>		<b>= <math>170 \text{ m}^2</math></b>

To find total surface area, both the large rink portion and the entrance must be added together, then subtract the area of overlap.

Area of overlap (back of the entrance)	= $5\text{m} \times 10\text{m}$	= <b><math>50 \text{ m}^2</math></b>
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**Total surface area** =  $6225 \text{ m}^2 + 170 \text{ m}^2 - 50 \text{ m}^2 = 6345 \text{ m}^2$

- b) The areas of the window, door and roof must be subtracted from the total surface area.

Area door	= $1\text{m} \times 2\text{m}$	= $2\text{m}^2$
Area of window	= $4\text{m} \times 2\text{m}$	= $8\text{m}^2$
Areas of roofs	= $2925 \text{ m}^2 + 60 \text{ m}^2$	= $2985 \text{ m}^2$

Area to be painted =  $6345 \text{ m}^2 - 2\text{m}^2 - 8\text{m}^2 - 2985 \text{ m}^2 = \mathbf{3350 \text{ m}^2 \text{ to be painted}}$

c) How many cans of paint are needed if each can covers  $300 \text{ m}^2$ ?

$$3350 \text{ m}^2 \div 300 \text{ m}^2 = 11.2 \text{ cans}$$

12 cans would be needed since 0.2 cans cannot be bought.

$$12 \text{ cans} \times \$45 = \$540.00$$

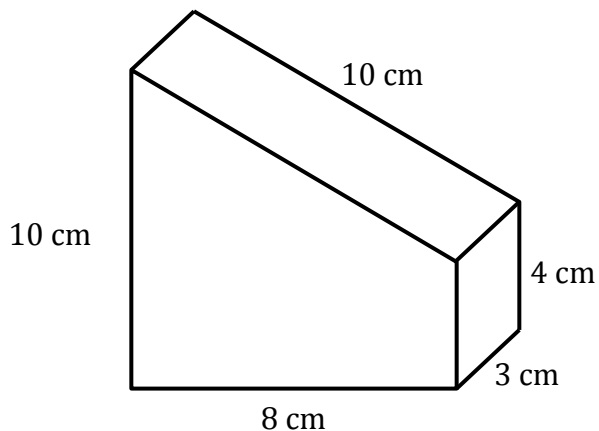
It would cost \$540.00 to paint the hockey rink.

### Sec 1.4 Surface Areas of Other Composite Objects

When determining the surface area of composite objects, we must determine what shapes make up the whole object and consider the overlap.

**Example 1** (from textbook p.34)

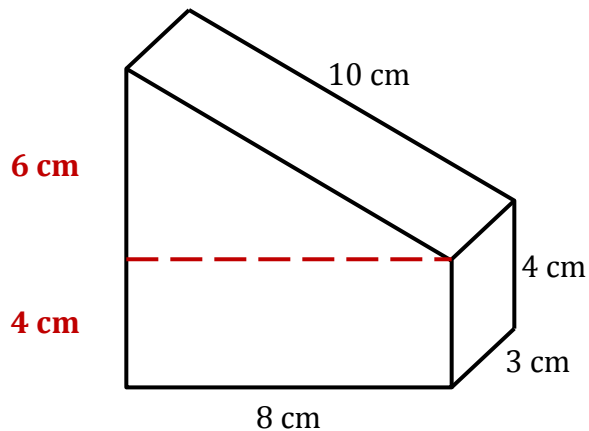
Determine surface area of this object.



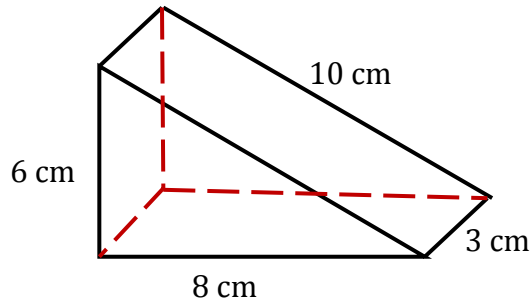
**Step 1:** What objects make up this whole object?

- ↳ a triangular prism
- ↳ a rectangular prism

**Step 2:** Find the surface area of each object.



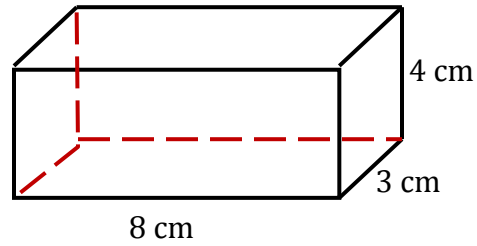
Triangular Prism



↳ surface area of triangular prism

$$\begin{aligned}
 &= 2 \text{ triangles} + 3 \text{ different rectangles} \\
 &= 2 \left( \frac{b \times h}{2} \right) + (l \times w) + (l \times w) + (l \times w) \\
 &= 2 \left( \frac{8 \times 6}{2} \right) + (8 \times 3) + (6 \times 3) + (3 \times 10) \\
 &= 48 + 24 + 18 + 30 \\
 &= 120 \text{ cm}^2
 \end{aligned}$$

Rectangular Prism



↳ surface area of rectangular prism

$$\begin{aligned}
 &= 2 \times \text{front} + 2 \times \text{right side} + 2 \times \text{top} \\
 &= 2(l \times w) + 2(l \times w) + 2(l \times w) \\
 &= 2(8 \times 4) + 2(4 \times 3) + 2(3 \times 8) \\
 &= 64 + 24 + 48 \\
 &= 136 \text{ cm}^2
 \end{aligned}$$

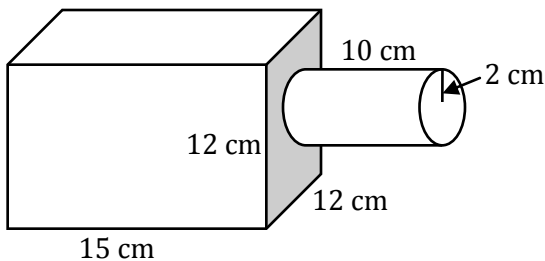
**Step 3:** Find the area of the overlap. Don't forget to double it!

$$\begin{aligned}
 \text{Overlap} &= (l \times w) \\
 &= (8 \times 3) \\
 &= 24 \times 2 = 48 \text{ cm}^2
 \end{aligned}$$

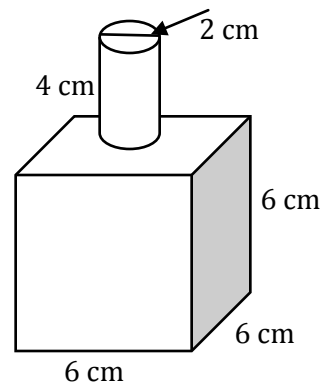
$$\begin{aligned}
 \text{Total surface area} &= \text{SA of triangular prism} + \text{SA of rectangular prism} - \text{overlap} \\
 &= 120 + 136 - 48 \\
 &= 208 \text{ cm}^2
 \end{aligned}$$

### Examples

1. Find the surface area of this object.



2. Find the surface area of this object.



Answers:

**1. Surface Area of Rectangular Prism**

$$\text{Front, Back, Top, Bottom} = 4 (12 \times 15) = 720 \text{ cm}^2$$

$$\text{Left, Right} = 2 (12 \times 12) = 288 \text{ cm}^2$$

$$\text{Total} : 1008 \text{ cm}^2$$

**Surface Area of Cylinder**

$$\text{Top, Bottom} = 2 \times \text{Area of circle} = 2 (\pi r^2) = 2 \times \pi \times 2^2 = 25.12 \text{ cm}^2$$

$$\text{Curved Surface} = 2\pi r \times h = 2 \times \pi \times 2 \times 10 = 125.6 \text{ cm}^2$$

$$\text{Total: } 150.72 \text{ cm}^2$$

**Area of Overlap**

Area of a Circle ....don't forget to double it!

$$2 \times \text{Area of circle} = 2 (\pi r^2) = 2 \times \pi \times 2^2 = 25.12 \text{ cm}^2$$

**Total Surface Area of the Composite Object**

SA of rectangular prism + SA of cylinder – area of overlap

$$1008 + 150.72 - 25.12 = 1133.6 \text{ cm}^2$$

**2. Surface Area of Rectangular Prism**

$$\text{Front, Back, Top, Bottom, Left, Right} = 6 (6 \times 6) = 216 \text{ cm}^2$$

$$\text{Total} : 216 \text{ cm}^2$$

**Surface Area of Cylinder**

$$\text{Top, Bottom} = 2 \times \text{Area of circle} = 2 (\pi r^2) = 2 \times \pi \times 1^2 = 6.28 \text{ cm}^2$$

$$\text{Curved Surface} = 2\pi r \times h = 2 \times \pi \times 1 \times 4 = 25.12 \text{ cm}^2$$

$$\text{Total: } 31.4 \text{ cm}^2$$

**Area of Overlap**

Area of a Circle ....don't forget to double it!

$$2 \times \text{Area of circle} = 2 (\pi r^2) = 2 \times \pi \times 1^2 = 6.28 \text{ cm}^2$$

**Total Surface Area of the Composite Object**

SA of rectangular prism + SA of cylinder – area of overlap

$$216 + 31.4 - 6.28 = 241.12 \text{ cm}^2$$

