

Lesson #4: Order of Operations of Integers

In lessons #1 and #2 you learned how to add, subtract, multiply and divide integers. Now you will learn to extend your ability to evaluate expressions containing positive and negative integers.

Before learning to evaluate using the order of operations you must know how to apply an exponent to an integer. An exponent is a base (integer) raised to a specific power.

For example 3^2 is an exponent. The base is the number 3 and the power is the number 2. This means to **multiply the base by itself two times**.

$$3^2 = 3 \times 3 = 9$$

Evaluate the following:

Exponential Form	Expanded Form	Simplified
3^2	3×3	9
2^3		
5^2		
2^1		

When applying the order of operations the acronym BEDMAS can be used to remember the order in which to perform the operations.

1. First any expression within **Brackets** must be performed. Brackets may be inside of brackets, in which case the innermost brackets must be calculated first.
2. Secondly any **Exponents** must be calculated.
3. Thirdly any **Division** or **Multiplication** operations must be performed. These should be done as they appear from left to right.
4. Lastly and **Addition** or **Subtraction** operations must be performed. These also should be done as they appear from left to right.

Evaluate the expression $(2 - 5) - (5 - 9)$

$(2 - 5) - (5 - 9)$	Apply BEDMAS
$= (-3) - (-4)$	You must begin by solving within the brackets.
$= -3 + 4$	Use subtraction of integer rules to subtract these.
$= 1$	Find the solution by adding the integers.

Example 1: Evaluate the expression $(-4 - 3) - (-8 + 6)$.

$$\begin{aligned}
 &(-4 - 3) - (-8 + 6) = \\
 &(-7) - (-2) \quad \text{"ADD THE OPPOSITE"} \\
 &\quad \downarrow \\
 &(-7) + (+2) = \boxed{-5}
 \end{aligned}$$

Evaluate the expression $-(-2)^3 - (-3)(4)$.

$-(-2)^3 - (-3)(4)$	Apply BEDMAS.
$= -1(-8) - 1(-3)(4)$	Insert all invisible 1's to make the negatives simpler. Since there are no calculations within brackets you get to move on to evaluating the exponent. $(-2)^3 = (-2)(-2)(-2) = -8$
$= 8 - (-12)$	Multiply and Divide from left to right. DO NOT CONFUSE THE SUBTRACTION SIGN BETWEEN AS A MULTIPLICATION!
$= 8 + 12$	Use rules of subtracting integers to make this into a simpler expression.
$= 20$	Evaluate.

Example 2: Evaluate the expression

a) $(-3)^3 + 2(-4)^2$

$$\begin{aligned} &\downarrow \\ &(-27) + 2(16) = \\ &(-27) + 32 = +5 \end{aligned}$$

b) $-(2)^4 - (-2)^3 + (-2)^2 - (-2)$

$$\begin{aligned} &-(16) - (-8) + (4) - (-2) \\ &\text{ADD THE OPPOSITE} \rightarrow \\ &-16 + (+8) + 4 + (+2) \\ &-16 + (+14) = -2 \end{aligned}$$

Evaluate the expression $\frac{-25 - (-13)}{4 - (-2)}$.

$\frac{-25 - (-13)}{4 - (-2)}$	Apply BEDMAS. Beware of "Invisible Brackets" that exist around the entire numerator and entire denominator.
$= \frac{[-1(25) - 1(-13)]}{[4 - 1(-2)]}$	Insert "Invisible Brackets" and "Invisible 1's"
$= \frac{[-25 + 13]}{[4 + 2]}$	Perform any multiplications within Brackets (once inside the brackets BEDMAS must be applied again)
$= \frac{-12}{6}$	Add and subtract within the top brackets.
$= -2$	Divide the integers.

Example 3: Evaluate the expression

a) $\frac{12 - (-16)}{-7 - (-3)}$ ADD THE OPPOSITE

$$\begin{aligned} &\frac{12 + (+16)}{-7 + (+3)} = \\ &\frac{28}{-4} = -7 \end{aligned}$$

b) $\frac{2^3 - (-2)^3}{7 - (-1)^3}$

$$\begin{aligned} &\frac{8 - (-8)}{7 - (-1)} = \\ &\frac{8 + (+8)}{7 + (+1)} = \frac{+16}{+8} = +2 \end{aligned}$$