

Unit 7.4

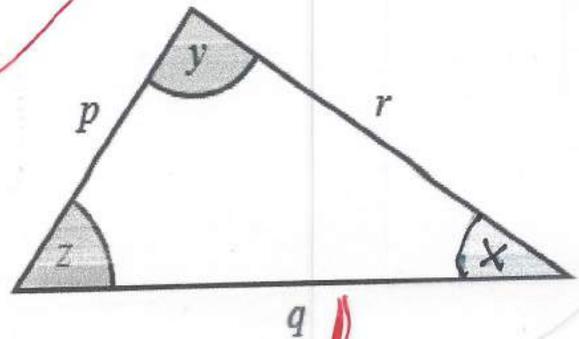
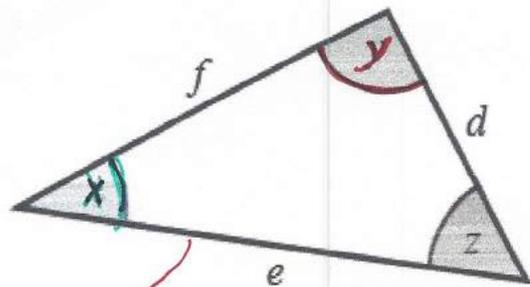
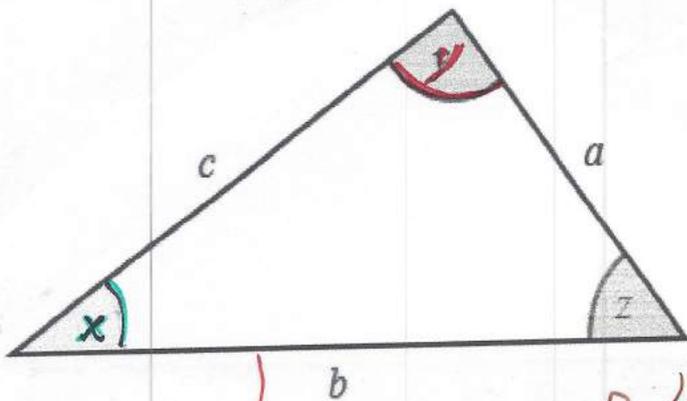
Similar Triangles

Similar Triangles:

- Same shape but NOT same size
- Corresponding pair of angles is equal
- The Ratio of any pair of corresponding sides is the same

↳ SAME SCALE FACTOR

Look at the following:

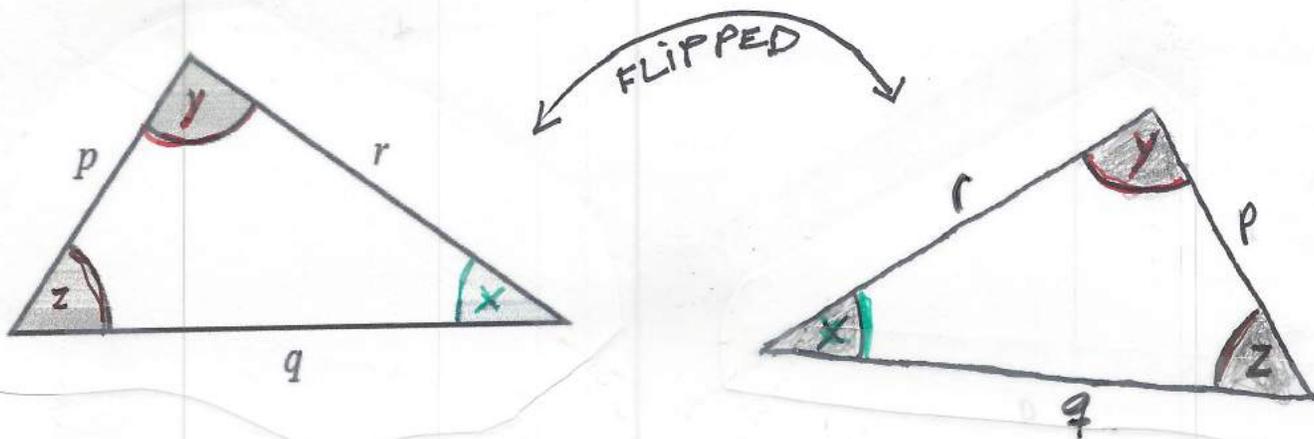
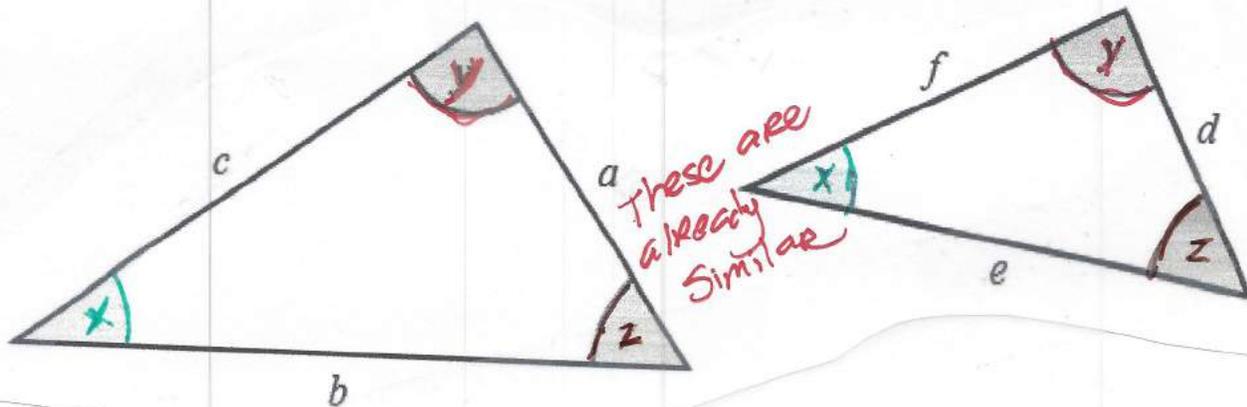


These two are SIMILAR

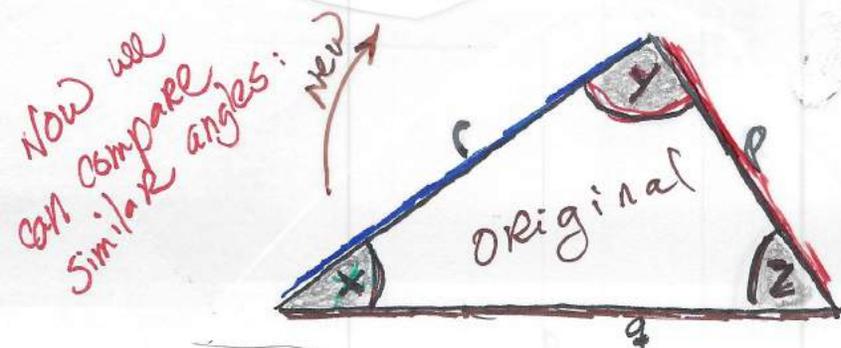
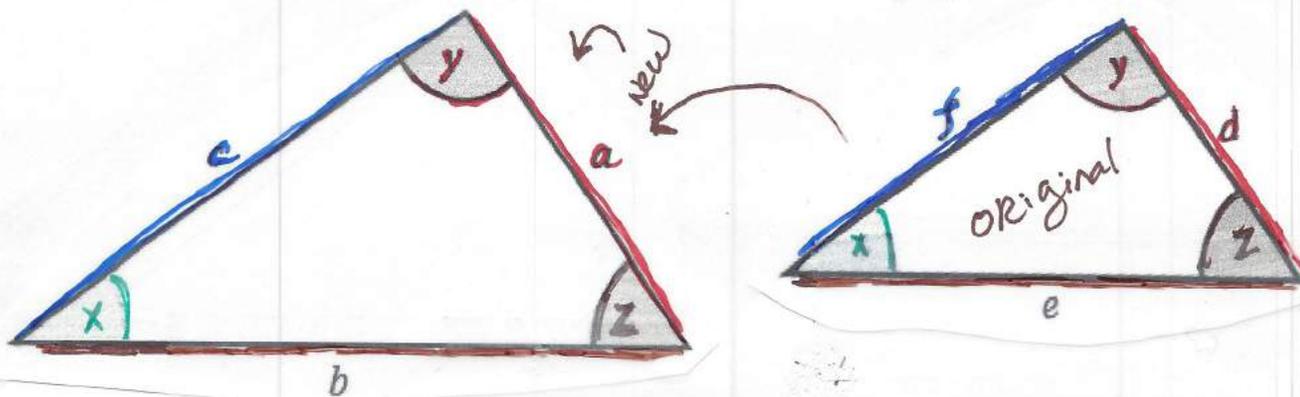
Notice that same angles, and the triangle itself are facing SAME WAY

In order for this triangle to be similar, it MUST BE FLIPPED

• Let's flip the third triangle



Now, all 3 are facing the same way:



$$\frac{a}{d} = \frac{c}{f} = \frac{b}{e}$$

$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$$

So, similar Triangles have

EITHER

→ Same angles →

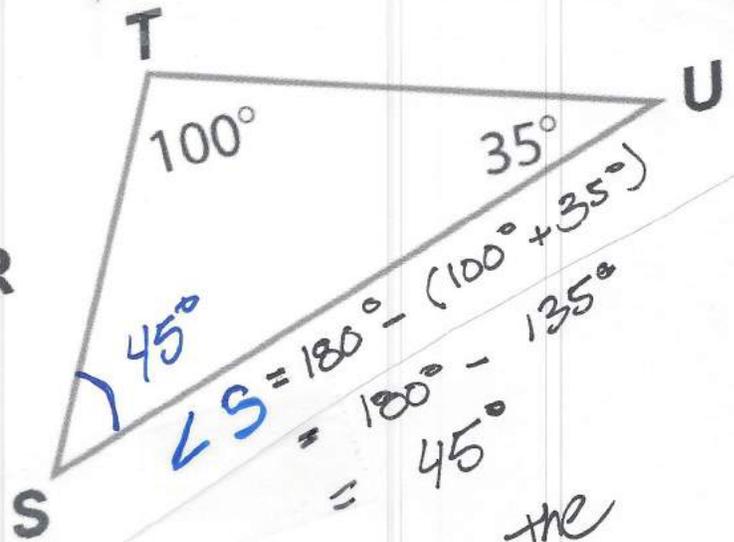
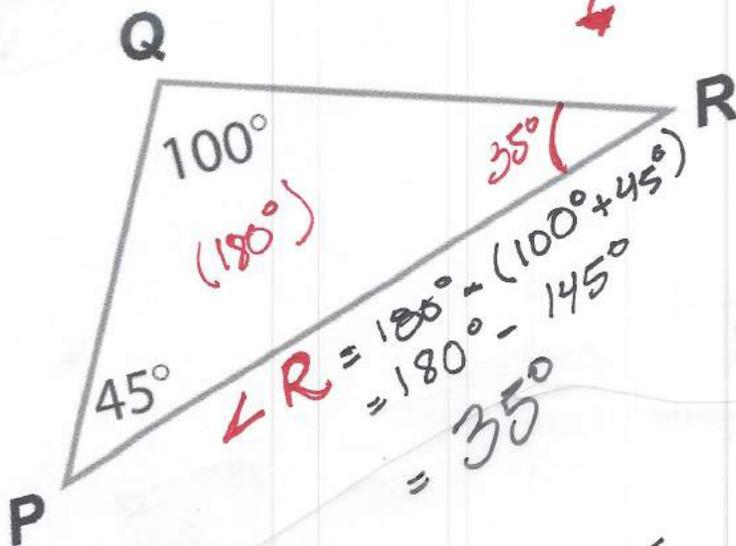
OR

→ Same Ratio of corresponding Sides →

Look at this Examples

1) Are the following triangles similar?

since angles are given, compare



Since $\angle Q = \angle T$
 $\angle P = \angle S$
 $\angle R = \angle U$

⇒ Triangles have the same angles

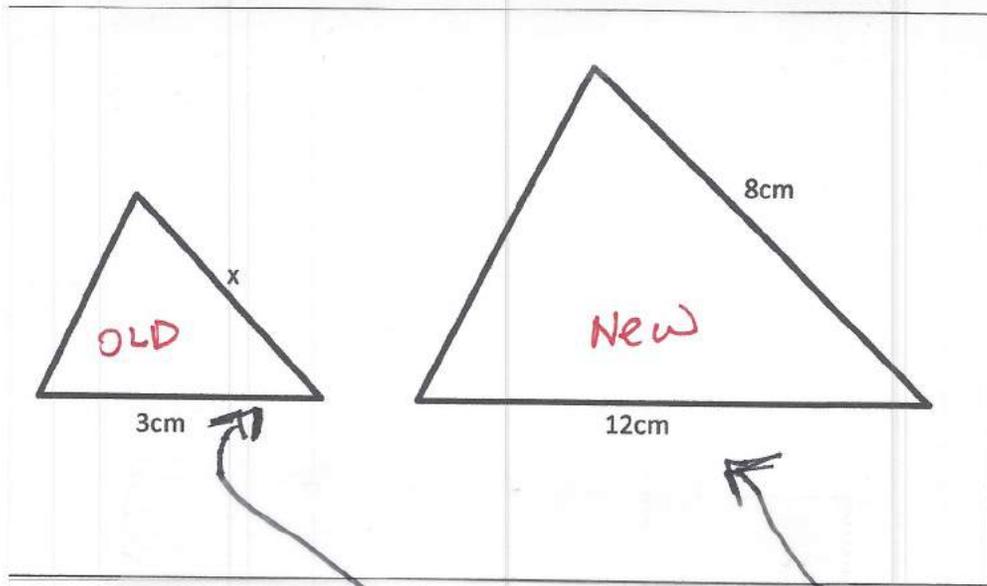
They are similar !!

2) These triangles are similar. Find x .

- this means angles are equal
- make sure the triangles have the same orientation
- corresponding sides have SAME **SCALE FACTOR**

Remember:

$$\text{SCALE FACTOR} = \frac{\text{New}}{\text{Old}}$$



Always
Find the
two
corres-
ponding
sides

• Scale Factor = $\frac{12 \text{ cm}}{3 \text{ cm}} = 4$

- Use S.F to calculate $x \Rightarrow$ Multiply the known side by S.F

However:

$$8 \text{ cm} \times \text{SF} = x$$

$$\text{If } \text{SF} = 4 \Rightarrow 8 \times 4 = 32 \text{ cm}$$

not possible!
it has to be smaller...
so

$$\text{SF} = \frac{1}{4} \quad \text{so}$$

$$x = 8 \times \frac{1}{4} = \frac{8}{4} = 2$$

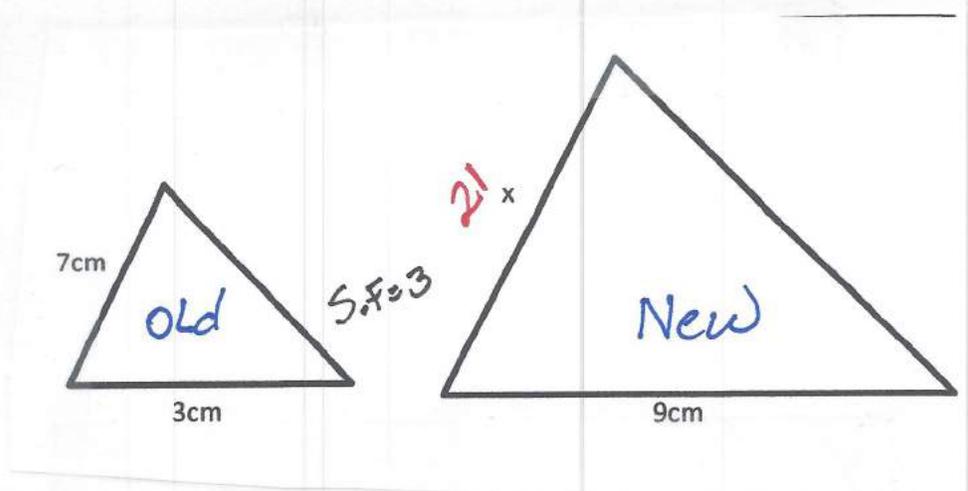
• OR ... use CROSS-multiplication Ratios:

$$\frac{12}{3} \quad \frac{8}{x} \Rightarrow \cancel{12} \cdot x = 8 \cdot \cancel{3}$$

$$\frac{\cancel{12}x}{\cancel{12}} = \frac{24}{12}$$

$$x = 2$$

3)



• Use Ratio ... cross-multiply:

$$\frac{9}{3} = \frac{x}{7} \Rightarrow 9 \cdot 7 = 3 \cdot x$$

$$x = \frac{9 \cdot 7}{3} = \frac{63}{3}$$

OR
• calculate S.F., then multiply

$$S.F. = \frac{\text{New}}{\text{Old}} = \frac{9 \text{ cm}}{3 \text{ cm}} = 3$$

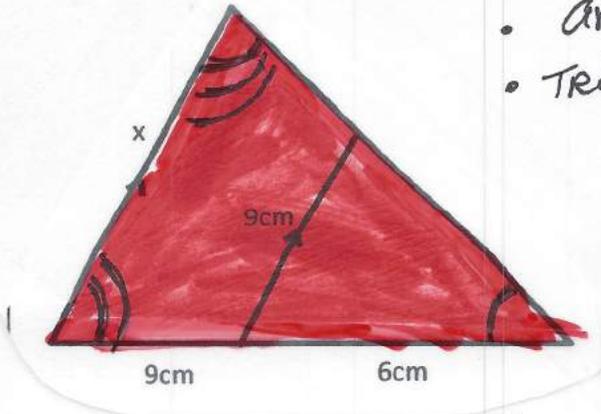
then

$$7 \times 3 = 21 \text{ cm} = x$$

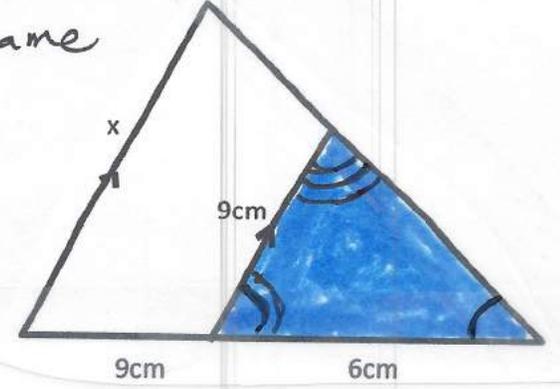
$$x = 21$$

This makes sense because it is an enlargement

4) Try this: they are similar triangles

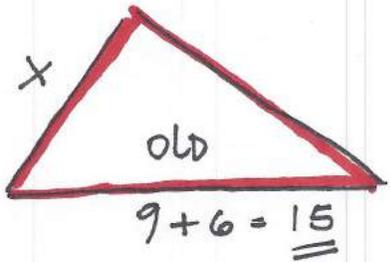


- angles are equal
- Triangles facing same direction

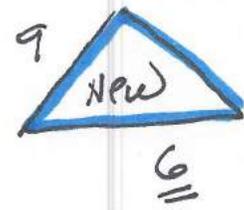


This is the first triangle =

This is the second triangle =



bigger to smaller



Now use Ratios

↳ use the two known corresponding sides

$$\frac{6}{15} \times \frac{9}{x} \Rightarrow \frac{6 \cdot x}{6} = \frac{15 \cdot 9}{6}$$

Note

$$x = \frac{15 \cdot 9}{6} = 22.5$$

• $SF = \frac{6}{15} = \frac{2}{5}$

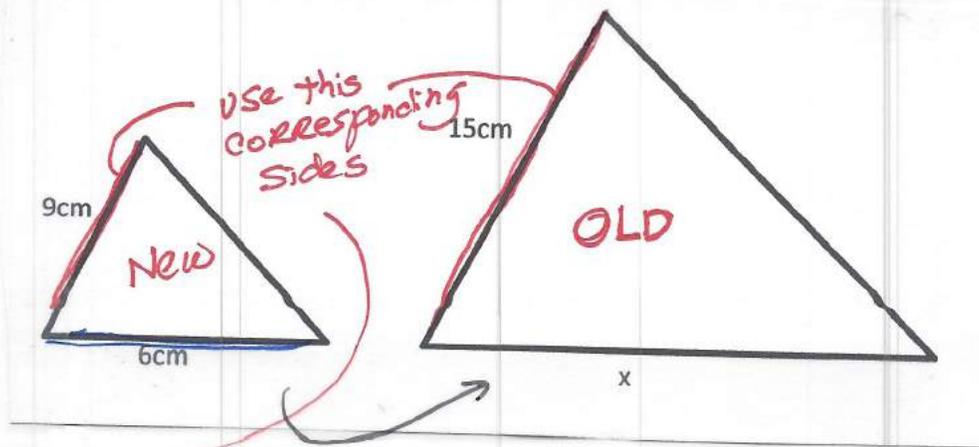
• $x = 9 \times S.F$

↳ $x = 9 \times \frac{2}{5} = \frac{18}{5} = 3.6$

this can't be because ~~x~~ should be bigger than 9

↳ SO $x = 9 \times \frac{5}{2} = \frac{45}{2} = 22.5$

3) Try assigning differently: (Make it a Reduction)



→ Scale factor = $\frac{\text{New}}{\text{Old}} = \frac{9}{15} = \frac{3}{5}$

• BUT since we are going backwards, then use $\frac{5}{3}$

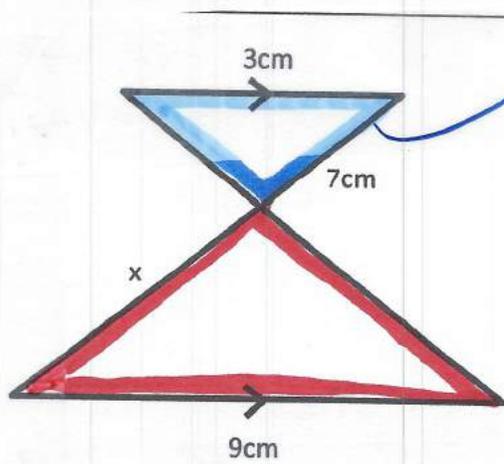
$$x = 6 \times \frac{5}{3} = \frac{30}{3} = 10$$

• OR Use Ratio:

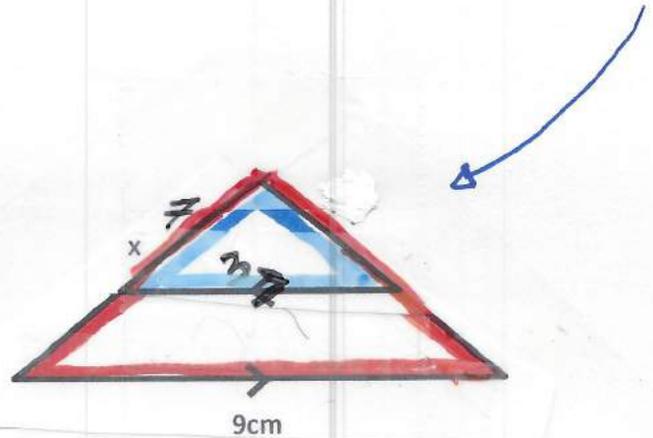
$$\frac{9}{15} \rightarrow \frac{6}{x} \Rightarrow \frac{9 \cdot x}{9} = \frac{15 \cdot 6}{9}$$

$$x = \frac{90}{9} = 10$$

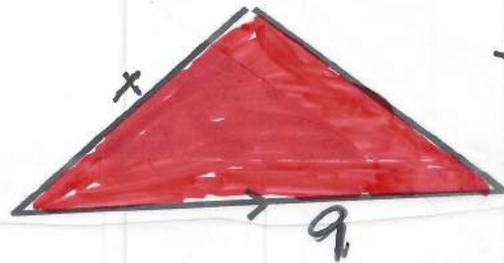
5) Remember to Re-arrange triangles so that they "look alike", OR have same orientation:



Flip this. Turn it upside down so that they end up inside of one another.

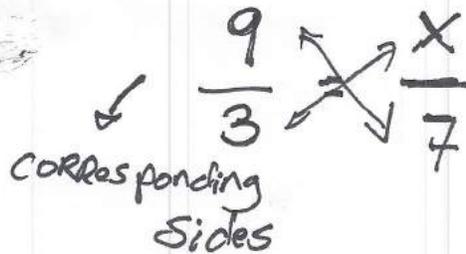


This is the first triangle



triangle 2 (New)

• Use Ratios -
CROSS-MULTIPLY:



$$\Rightarrow \frac{3x}{3} = \frac{9 \cdot 7}{3}$$

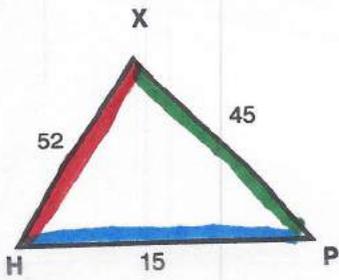
$$x = \frac{63}{3} = 21$$

• Calculate SF =

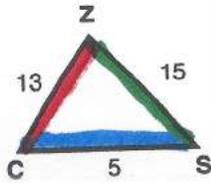
$$\frac{\text{New}}{\text{Old}} = \frac{9}{3} = 3$$

• to get x $\Rightarrow x = 7 \cdot SF = 7 \cdot 3 = 21$

LOOK AT THE SIMILARITY STATEMENTS



old



New

- Angles are equal maybe?
- Since they are similar, they must have the same scale factor

Similarity statements
(corresponding sides)

$$\begin{aligned} \angle X &= \angle Z \\ \angle H &= \angle C \\ \angle P &= \angle S \end{aligned}$$

$$\frac{CS}{HP} = \frac{ZS}{XP} = \frac{ZC}{XH}$$

Labels: BLUE scale factor (under CS/HP), RED scale factor (under ZS/XP), Green scale factor (under ZC/XH)

$$\frac{5}{15} = \frac{15}{45} = \frac{13}{52}$$

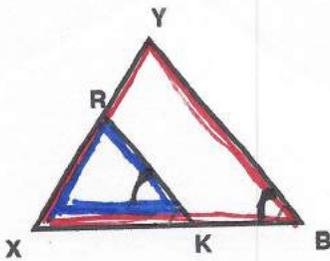
↓	↓	↓
0. $\bar{3}$	0. $\bar{3}$	0.25

See how the scale factors are different? then

NOT similar

($\triangle XPH \neq \triangle ZSC$)

7) IF THESE TRIANGLES ARE SIMILAR,
 WRITE THE SIMILARITY STATEMENTS



Similar!

• We can see that
 $\angle B = \angle K$
 and that they share one
 angle

$$\angle X = \angle X$$

• this means that the other
 angle is also equal

$$\angle Y = \angle R$$

So Angle Similarity Statement:

$$\angle B = \angle K, \angle X = \angle X, \angle Y = \angle R$$

What are the corresponding sides?
 Write the scale factor for each pair:

YB and RK are similar

XY and XR are similar

XB and XK are similar

Make it an enlargement so
 that $\triangle XYB$ is "new" and
 $\triangle XRK$ is old

$\frac{\text{New}}{\text{old}}$

$$\frac{XY}{XR} = \frac{YB}{RK} = \frac{XB}{XK}$$