

Solving Equations Using Inverse Operations

Inverse Operations:

Operation	Inverse Operation
Addition +	Subtraction -
Subtraction -	Addition +
Multiplication *	Division ÷
Division ÷	Multiplication *

1st LINE → "BUILD THE EQUATION"
 2nd LINE → "SOLVE THE EQUATION"

EXAMPLE: $\{2x + 5 = 15\}$ → "ANSWER"

Line 1
 "Build the equation"
 EQUATION

x $2x$ $2x+5$

\downarrow $\curvearrowleft \times 2$ $\downarrow +5$

ALWAYS
 START WITH
 VARIABLE
 HERE

the inverse
 of $+5$
 -5

ALWAYS : In
 here you
 START WITH
 THE "ANSWER"

Line 2
 "Solves the
 equation.."

• START AT THE
 END, WORK
 BACKWARDS
 USING INVERSE OPERATIONS!

5 10 15

$\downarrow \div 2$ $\downarrow -5$ \downarrow

the inverse
 of $\times 2$
 $\div 2$

the inverse
 of $+5$
 -5

THIS IS
 YOUR ANSWER!

$x=5$

check it:

$2(5) + 5 = 15$

it works!

Practice 3.1

Solve each equation.

$$1) 10x = 150$$

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Using inverse
operations

$$2) 8 + x = 9$$

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$$3) v - 11 = 7$$

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<input type="text"/>	<input type="text"/>

$$4) -4 = -2n$$

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<input type="text"/>	<input type="text"/>

$$13) -15 = -4m + 5$$

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$$14) 10 - 6v = -104$$

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$$15) 8n + 7 = 31$$

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$$16) -9x - 13 = -103$$

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So that you know,
these equations are called
"2-step Equations"

(Because it takes 2 steps to solve them)

Now, Let's Solve Equations that have variables on both sides

Example →

$x+4 = 2x - 6$	x is on both sides	Original Problem.	<ul style="list-style-type: none"> Move one variable so that both variables are on the same side. Combine the variables Now solve as usual.
$x+4 - 4 = 2x - 6 - 4$	 move the $2x$ to the side of the x	Let's start by trying to get all of the constants on the right hand side of the equation. I am going to subtract 4 from BOTH sides so that x is left on the left hand side.	
$x = 2x - 10$	 move	Simplify. $-6 - 4 = -10$ on the right hand side. But we still have $2x$ to get on the left hand side.	
$x - 2x = 2x - 2x - 10$	 Combine the x 's	I subtracted $2x$ from BOTH sides, so now no variable terms are left on the right and $x - 2x$ is on the left hand side.	
$-x = -10$	Anytime the variable is negative, change all signs	We want our answer to be a positive x not a negative x .	
$(-1)(-x) = -10(-1)$	 it is the same thing as multiplying by -1	Multiply both sides by -1 to make x positive on the left hand side.	
$x = 10$		Our final answer.	
Check: $x+4 = 2x - 6$ $10 + 4 = 2(10) - 6$ $14 = 14$		Substitute 10 for x in the equation. Since both sides are equal to 14, our answer $x = 10$ is correct.	

Solving Multi-Step Equations

Variables on Both Sides - Negative Coefficients

Name: _____ Date: _____



Solve the equations.

$$(1) \quad -5x - 77 = x + 61$$

I'll move the $-5x$ combine
 $-5x + 5x - 77 = x + 5x + 61$
 $\cancel{-77} = \cancel{61} + 6x$
 now, get rid of $+61 \rightarrow -77 - \underline{61} = 6x + 61 - \underline{61} \Rightarrow -138 = 6x$

$$(3) \quad 4x - 35 = 31 + 7x$$

$$21 - 148 = x - 42 + 3x$$

$$\frac{-138}{6} = \frac{6x}{6}$$

$$x = -23$$

$$(4) \quad -3x - 21 = -x + 39$$

$$(5) \quad 3x - 177 = 12x + 48$$

$$(6) \quad 8x - 10 = 13 + 7x$$

$$(7) \quad -111 - 3x = 92 + 4x$$

$$(8) \quad 7x - 37 = 23 + 4x$$

$$(9) \quad 8x - 93 = 111 - 4x$$

$$(10) \quad -14 - 5x = 32 - 3x$$

$$(11) \quad 5x - 11 = 15 + 7x$$

$$(12) \quad -69 + 5x = 81 + 11x$$

Solving Equations with Fractions

Remember: what you do to one side, you do to the other side!

$$\frac{2x}{5} - 4 = 8$$

• FIRST, Eliminate the -4 on the left so that only the fraction with the variable is left

$$\frac{2x}{5} - 4 + 4 = 8 + 4$$

$$\frac{2x}{5} = 12$$

• Now, eliminate the 5 that is dividing

$$\left(\frac{2x}{5}\right) \times 5 = 12 \times 5$$

$$2x = 60$$

Now, eliminate the 2 that multiplies x

$$\frac{2x}{2} = \frac{60}{2}$$

$$x = 30$$

Check it!
with original equation

$$\frac{2(30)}{5} - 4 = \frac{60}{5} - 4$$

Now what if the variable is in the denominator?

$$= 12 - 4 = 8$$

It checks!

$$\frac{2}{5x} - 10 = 4$$

• To eliminate the $5x$ on the bottom (which is dividing), multiply by $5x$ (both sides)

$$\frac{2}{5x} - 10 + 10 = 4 + 10$$

But first, move the -10

$$\frac{2}{5x} = 14$$

$$\left(\frac{2}{5x}\right)5x = 14 \times 5x$$

$$\begin{aligned} 2 &= 70x && \cdot \text{Divide by } 70 \\ \frac{2}{70} &= \frac{70x}{70} && \boxed{x = \frac{2}{70}} \end{aligned}$$

- In Reality, you are doing exactly the same thing you have been doing.

Goal \rightarrow Variable alone, with 1 in front, by itself and in the Numerator

Solve each equation.

$$1) 6 = \frac{a}{4} + 2$$

$$2) -6 + \frac{x}{4} = -5$$

$$5) -4 = \frac{r}{20} - 5$$

$$6) -1 = \frac{5+x}{6}$$

$$3) \frac{a}{4} = \frac{15}{4}$$

$$7) \frac{v+9}{3} = 8$$

$$5) 15 = \frac{v}{2}$$

$$11) -2 = 2 + \frac{v}{4}$$

$$\frac{122}{10x} - 2 = 12$$

$$\frac{0.97}{x} + 3.5 = 6.4$$

$$\frac{36}{x} = 36$$

$$\frac{56}{a} = -3.5$$

Name _____ Date _____

Distributive Property

Solving Equations Using The Distributive Property

The distributive property allows us to remove the parentheses by distributing the value outside the parentheses with each term located inside the parentheses. The distributive property is a way to write an expression in expanded form.

Example: $3(6 + 9)$

There are two terms inside the parentheses.

$$\overbrace{3(6 + 9)}$$

Multiply the 3 by each term inside the parentheses.

$$3 \times 6 + 3 \times 9$$

Rewrite

$$18 + 27 = 45$$

Solve

Whenever a number is being multiplied by each term inside a set of parentheses it should be recognized as the distributive property. The distributive property is necessary to solve some algebraic equations or to simplify some algebraic expressions.

Example: Solve $5(x - 8) = 10$

$$\overbrace{5(x - 8)}$$

$$= 10$$

$$b) 5x - 40 = 10$$

$$+40 +40$$

$$c) \frac{5x}{5} = \frac{50}{5}$$

$$d) x = 10$$

Directions: Find the value of x for each of the following equations.

$$1) 6(x + 3) = 48$$

$$2) 4(5 - x) = 8$$

$$3) 7(x + 2) + 3 = 73$$

$$4) 8(x - 3) = 96$$

$$6x + 18 = 48$$

$$6x + 18 - 18 = 48 - 18$$

$$\begin{array}{r} \cancel{6}x = \frac{30}{6} \\ \boxed{x = 5} \end{array}$$

$$5) 9(x + 3) = 27$$

$$6) 12(x + 20) = 372$$

$$7) 4(x - 5) = 44$$

$$8) 3(x - 20) = 15$$

$$9) 5(x + 9) - 4 = 51 \quad 10) 8(x + 100) - 3 = 837 \quad 11) 2(x - 5 + 2) = 6 \quad 12) 20(x + 1) = 200$$

$$13) 12(x - 4) = 144$$

$$14) 9(x + 9) = 90$$

$$15) 7(2 + x) = 35$$

$$16) 8(11 - x) = 16$$

More Practice

$$13) -15 = -4m + 5$$

$$14) 10 - 6v = -104$$

$$15) 8n + 7 = 31$$

$$16) -9x - 13 = -103$$

$$17) \frac{n+5}{-16} = -1$$

$$18) -10 = -10 + 7m$$

$$19) -10 = 10(k - 9)$$

$$20) \frac{m}{9} - 1 = -2$$

$$21) 9 + 9n = 9$$

$$22) 7(9 + k) = 84$$

$$23) 8 + \frac{b}{-4} = 5$$

$$24) -243 = -9(10 + x)$$

Extra Practice →

Use whichever
method you prefer!

Solving Linear Equations: The Ultimate Guidelines

The Preparation:

1. Eliminate any parentheses by distributing.
2. Clear any fractions or decimals by multiplying each side by the equation's LCD.
3. Combine like terms on each individual side.

Isolating the Variable:

4. If necessary, reduce the equation to standard form.
5. If necessary, isolate the variable term then finish solving.

Example 6: Solve each equation and verify the solution.

a) $\frac{1}{2}\left(x + \frac{2}{3}\right) = 3(x - 1)$

b) $0.2(3y - 5) = 0.15(2y + 3) - 0.85$

Procedure: First distribute, then clear the fractions or decimals.

Answer:

a) $\frac{1}{2}\left(x + \frac{2}{3}\right) = 3(x - 1)$

Distribute and simplify; $\frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$

$\frac{1}{2}x + \frac{1}{3} = 3x - 3$

Lower Common Denominator

The LCD is 6. Prepare the equation by placing parentheses around each side. Multiply each side by 6.

$\frac{6}{1}\left(\frac{1}{2}x + \frac{1}{3}\right) = 6(3x - 3) \quad \text{OR} \quad \frac{1 \times 3}{2 \times 3}x + \frac{1 \times 2}{3 \times 2} \Rightarrow \frac{3x}{6} + \frac{2}{6} = (3x - 3)$

Simplify: $\frac{6}{1} \cdot \frac{1}{2}x = 3x$ and $\frac{6}{1} \cdot \frac{1}{3} = 2$.

$3x + 2 = 18x - 18$

$\frac{3x+2}{6} = 3x - 3 \quad \text{OR}$

now you can add

Reduce this to standard form by adding $-3x$ to each side.

$3x + (-3x) + 2 = 18x + (-3x) - 18$

Simplify.

$(\frac{3x+2}{6}) \cancel{\times 6} = 6(3x-3)$

$2 = 15x - 18$

Isolate the variable term by adding +18 to each side.

$2 + 18 = 15x - 18 + 18$

Verify the solution $\frac{4}{3}$:

$20 = 15x$

Divide each side by 15.

$\frac{1}{2}\left(x + \frac{2}{3}\right) = 3(x - 1)$

$\frac{20}{15} = \frac{15x}{15}$

Simplify.

$\frac{1}{2}\left(\frac{4}{3} + \frac{2}{3}\right) = 3\left(\frac{4}{3} - 1\right)$

$\frac{4}{3} = x$

$\frac{1}{2}\left(\frac{6}{3}\right) = 3\left(\frac{4}{3} - \frac{3}{3}\right)$

$x = \frac{4}{3}$

$\frac{6}{6} = \frac{3}{1}\left(\frac{1}{3}\right)$

$1 = 1 \checkmark$

b)	$0.2(3y - 5) = 0.15(2y + 3) - 0.85$	Distribute.
	$0.6y - 1.0 = 0.30y + 0.45 - 0.85$	Write each decimal so that it has two decimal places.
	$0.60y - 1.00 = 0.30y + 0.45 - 0.85$	Prepare the equation by placing parentheses around each side. Multiply each side by 100.
	$100(0.60y - 1.00) = (0.30y + 0.45 - 0.85)100$	
	$60y - 100 = 30y + 45 - 85$	Combine like terms on the right side.
	$60y - 100 = 30y - 40$	Reduce this to standard form by adding $-30y$ to each side.
	$60y + (-30y) - 100 = 30y + (-30y) - 40$	Simplify.
	$30y - 100 = -40$	Add 100 to each side.
	$30y - 100 + 100 = -40 + 100$	Add 100 to each side.
	$30y = 60$	Divide each side by 30.
	$\frac{30y}{30} = \frac{60}{30}$	
	$y = 2$	You finish it: Verify that 2 is the solution.