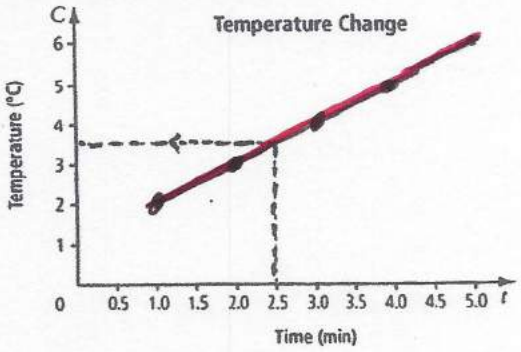
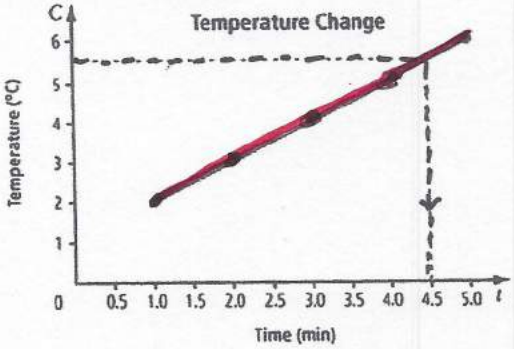
INTERPOLATION ↙

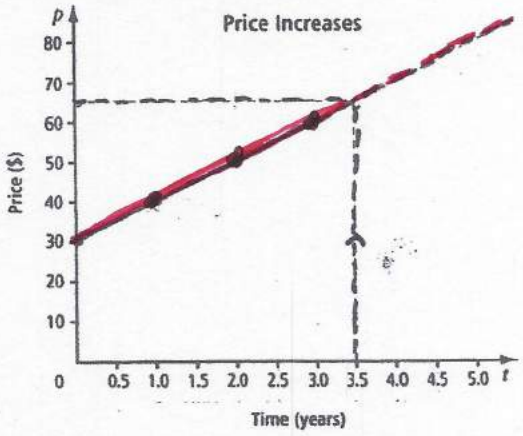
Key Ideas

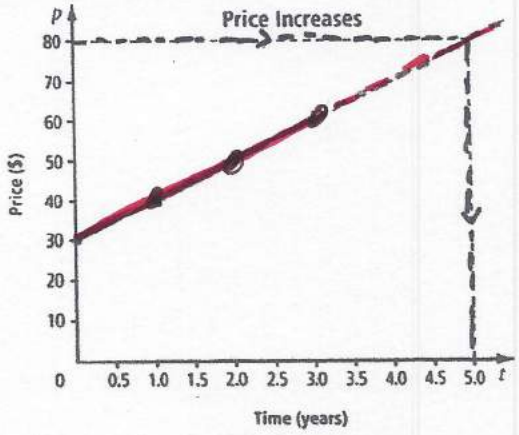
- On a graph, you can use a line to interpolate values between known values.
  - Start with a known value for x.
  - Start with a known value for y.





- On a graph, you can extend a line to extrapolate values beyond known values.
  - Use a dashed line to extend the line beyond the known x-value or y-value.
  - Start with a known value for x.
  - Start with a known value for y.





- Interpolation and extrapolation should be used only when it is reasonable to have values between or beyond the values on a graph.

EXTRAPOLATION ↗

4.5 Using Graphs to Estimate Values

Interpolation:

A prediction between given data points. This means that the graph does not have to be extended.

Extrapolation:

A prediction beyond the given data points. Because a linear relation is constant (x and y patterns), you can always "extend" the graph with a ruler

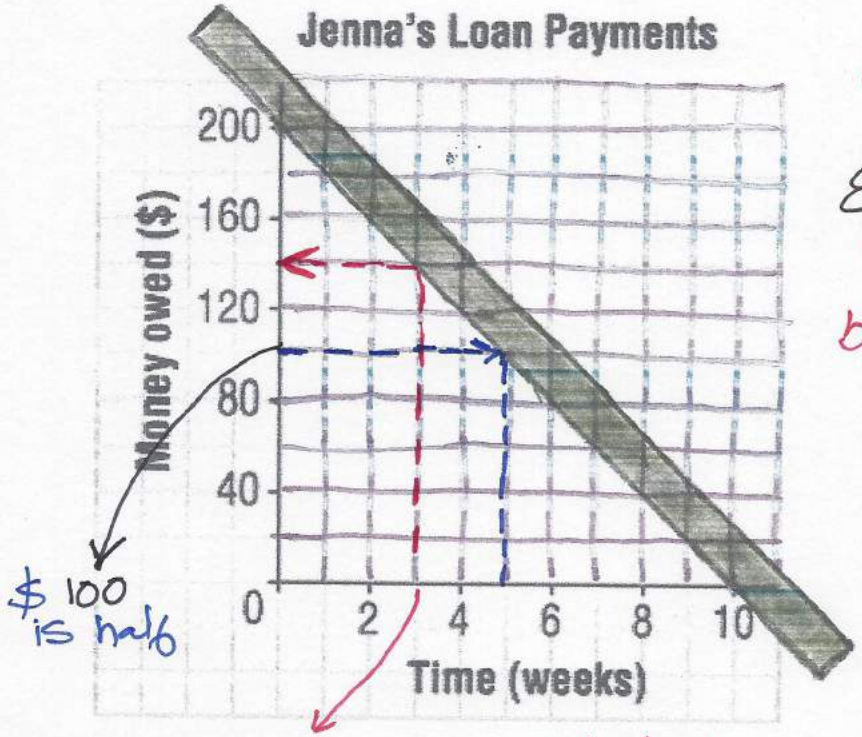
Examples:

1) Jenna borrows money from her parents for a school trip. She repays the loan by making regular weekly payments. The graph shows how the money is repaid over time. The data are discrete because payments are made every week.

a) How much money did Jenna originally borrow?

b) How much money does she still owe after 3 weeks?

c) How many weeks will it take Jenna to repay one-half of the money she borrowed?



a) the highest point on the y-axis is $200. this is how much she borrowed originally.

b) Find 3 weeks on the x-axis
- Go up towards the graph, then towards the y-axis.
- $140 are owed after 3 weeks. this means she still owes 140
  200 − 140 = 60 has paid

c) half the money is $100.
- starts at $100 (y-axis)
- go towards graph, then down
- 5 weeks

2)

Maya jogs on a running track. This graph shows how far she jogs in 10 min. Assume Maya continues to jog at the same average speed.

this means this is a linear relation

Use the graph.

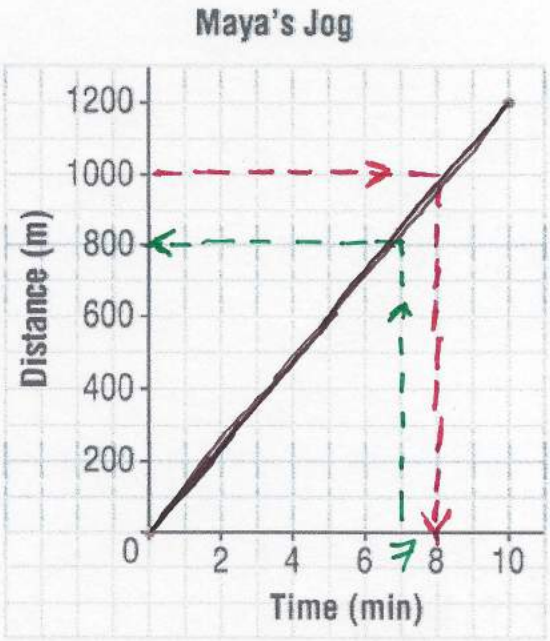
a) Predict how long it will take Maya to jog 2000 m.

b) Predict how far Maya will jog in 14 min.

c) What assumption did you make?

a) the y-axis "ends" at 1200 m. To find out how long it will take Maya to jog 2000 m, we must "extrapolate".

- Since her speed is constant, find out how much it takes her 1000 m. Start at 1000 m, go towards the graph, then down. It takes her 8 min to jog a 1000 m.
- this means 2000 m → 8 min × 2 = 16 min - 17 min



b) Again, we must extrapolate.

- Find out how far she jogs in 7 min.
- start at 7 min (x-axis). go up and towards the y-axis (distance).
- She jogs 800 m in 7 min.
- this means

14 min = (800 m) × 2 = 1600 m approx.

c) We have to assume that she keeps her speed constant.

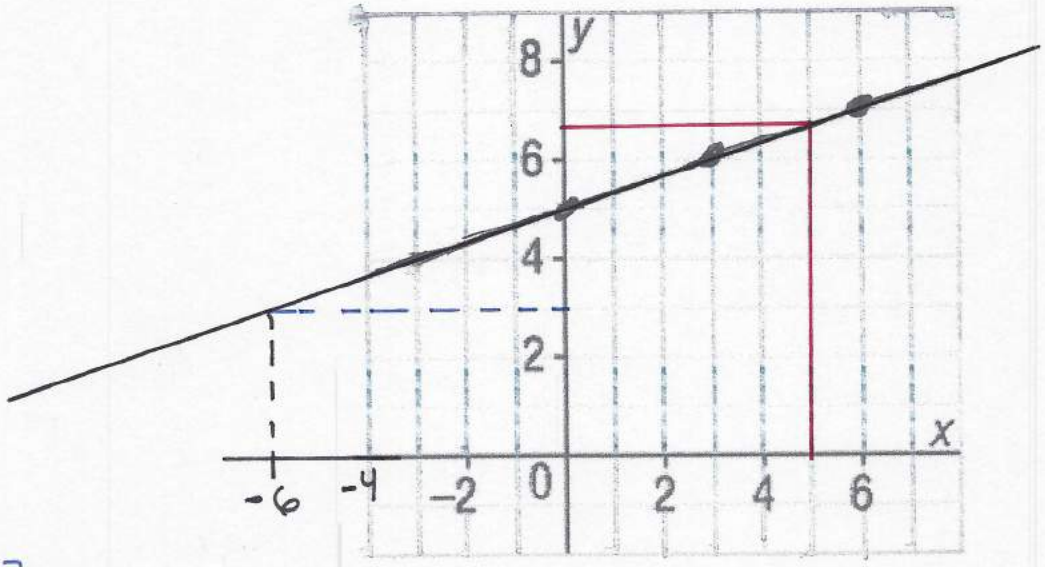
3)

Use this graph of a linear relation.

a) Determine the value of x when $y = 3$.

b) Determine the value of y when $x = 5$.

a) $x = ?$ when $y = 3$ ⇒
- Start at $y = 3$
- Notice that we must extend the graph
- Notice also how we must move towards the left.
- It looks like it is approx. -6



b) $y = ?$ when $x = 5$ →
- Start at $x = 5$
- go up, then to the left
- It looks like $\approx y = 6.5 \approx 6.75$

Let's make a table of values:

x	y
0	5
3	6
6	7

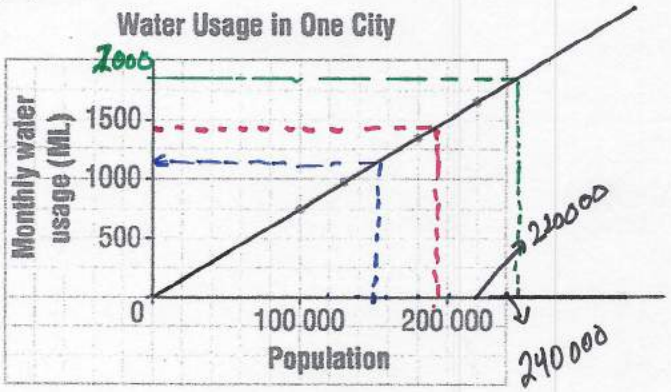
Assignment: Pg. 196-198 #4-8, 13 & 14

Set Activity:

A city has grown over the past few years. This table and graph show how the volume of water used each month is related to the population.

Population	Monthly Water Usage (ML)
100 000	750
130 000	975
180 000	1350
220 000	1650

1 ML is 1 000 000 L.



Use these data to:

- Estimate the monthly water usage for a population of 150 000 people.
- Estimate the population when the monthly water usage is 1400 ML.
- Predict the water usage for 250 000 people.

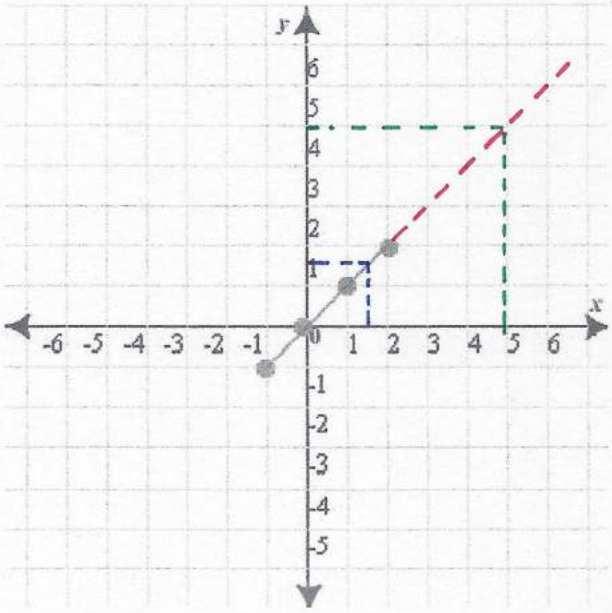
- Start at 150,000 on the x-axis
- Go up and to the left.
- This results in water usage between 1000 and 1250 ML

approximate 1400 mL on the y-axis.
this shows that 1400 ML is consumed by a population between 180 000 - 200 000.

this shows that for approx. 250 000 population, the usage is between 1750 and 2000 ML

7. A) Create a table from this graph.

x	y
-1	-1
0	0
1	1
2	2



B) Interpolate from the graph y if x = 1.5.

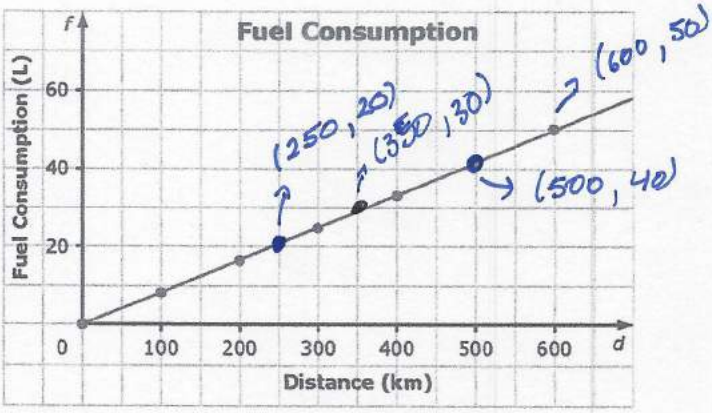
When $x=1.5$, $y=1.5$

C) Extrapolate from the graph y if x=5.

when $x=5$, $y=5$

TRY THESE PROBLEMS!

4. The graph shows the relationship between the fuel consumption, *f*, in litres (L), and the distance driven, *d*, in kilometres (km).



Make a Table of Values

x	y
0	0
250	20
500	40

a) What is the linear equation?

b) How far could you drive with 34 L of gas?

c) Is it appropriate to interpolate or extrapolate values on this graph? What assumption is being made? Explain.

a) $\frac{250}{20} = 12.5$ $\quad x = 12.5y \Rightarrow \boxed{y = \frac{x}{12.5}}$

b) $(34\ L) = \frac{x}{12.5}$ then $x = (34\ L)(12.5) = 425$ you can drive 425 km

c) It must be assumed that the fuel consumption is constant

21. The cost to print digital photos at an online store is shown in the graph below.

a. Extrapolate to estimate the cost of printing 60 photos

60 photos cost $12

b. Extrapolate to estimate the number of photos you could print for $1.00

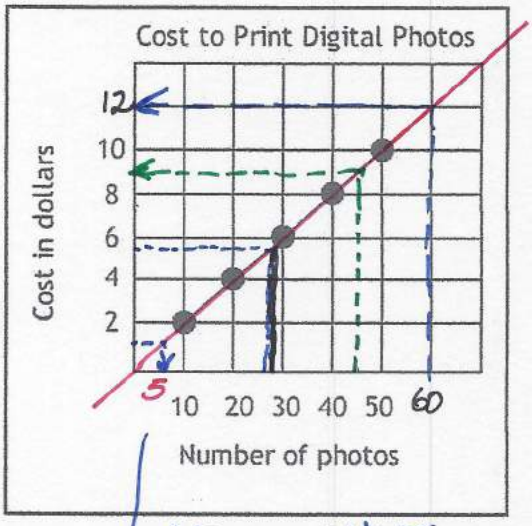
$1.00 pays for 5 photos

c. Interpolate to estimate the cost of printing 45 photos

45 photos cost ≈ $9.00

d. Interpolate to estimate the number of photos you could print for $5.50.

≈ 27-28 pictures



We see that:

10 → 2

$x = y \cdot 5 \Rightarrow \boxed{y = \frac{x}{5}}$

y = $ x = photos

22. The following graph shows the cost to rent the gym for 2, 4 or 6 hours at $30.00 per hour.

a. Extrapolate to find the cost to rent the gym for 8 hours.

8 hours → $240

b. Extrapolate to find how long you could rent the gym for $30.00.

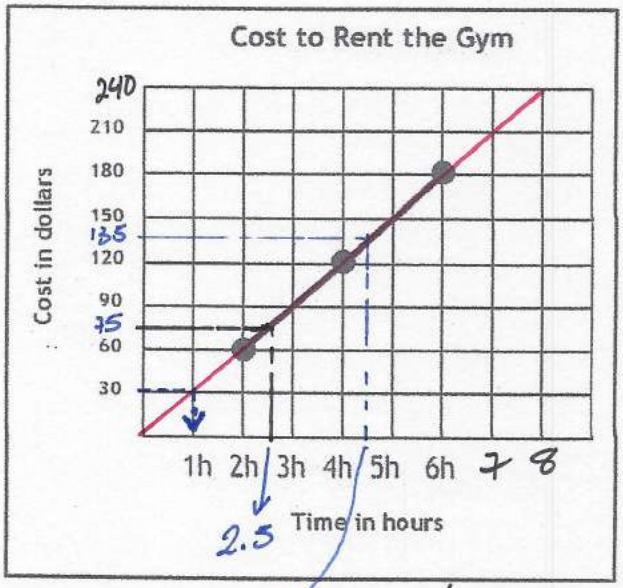
$30 → 1 hour

c. Interpolate to find cost to rent the gym for 2.5 hours

2.5 h → $ 75

d. Interpolate to find how long you could rent the gym for $135.00.

$135 → 4.5 h



x	y
1	30
2	60
3	90

)+30

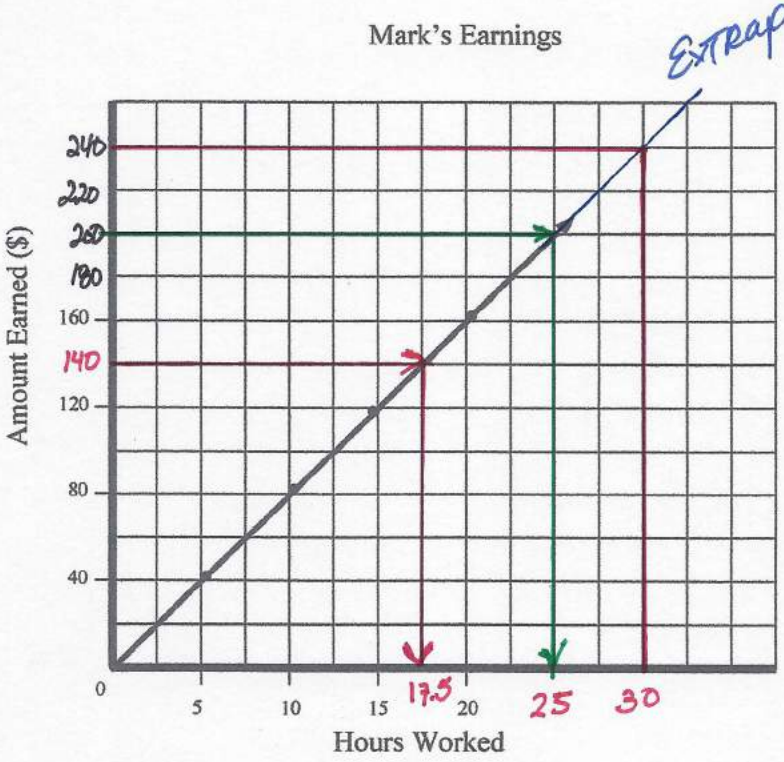
$\boxed{y = 30x}$

x → hours

y → $

Example 3 :

Use the graph to answer each of the following questions and state whether you used interpolation or extrapolation to find the answer.



Extrapolate

$y = ?x$

$40 = ?5$

$\boxed{y = 8x}$ $y \Rightarrow \$$, $x \Rightarrow$ hours

a) How many hours must Mark work to earn $140?

With equation

Approx. 17.5 hours

$140 = 8x$

$x = 17.5$

b) How much would Mark earn if he worked 30 hours?

30 hours → $ 240

$y = 8 \times 30$

$= 240\$$

c) How many hours did Mark work if he earned $200 last week?

$ 200 → 25 hours worked

$200\$ = 8x$

$x = 25$ hours

Extra Practice 5

Lesson 4.5: Using Graphs to Estimate Values

1. This graph represents a linear relation.

a) Determine the value of x for each value of y.

i) $y = 1$ $x=0$ ii) $y = 3$ $x=4$

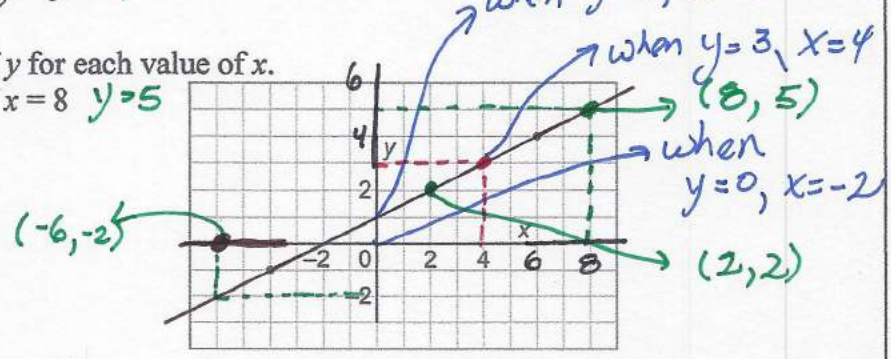
iii) $y = 0$ $x=-2$

b) Determine the value of y for each value of x.

i) $x = 2$ $y=2$ ii) $x = 8$ $y=5$

iii) $x = -6$ $y=-2$

x	y
0	-2
1	0
2	2
3	4



2. This graph represents a linear relation.

a) Determine the value of x for each value of y.

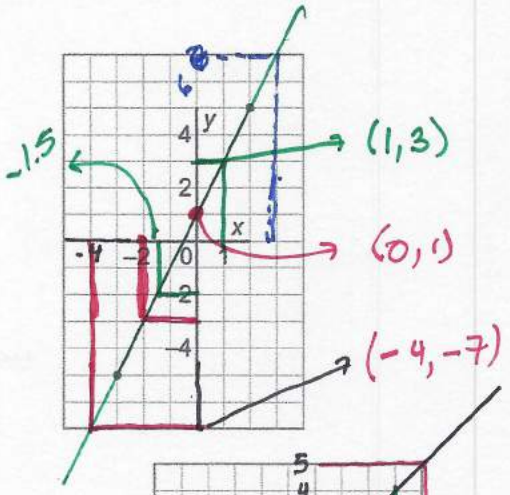
i) $y = 3$ $x=1$ ii) $y = -2$ $x=-1.5$

iii) $y = 7$ $x=3$

b) Determine the value of y for each value of x.

i) $x = 0$ $y=1$ ii) $x = -2$ $y=-3$

iii) $x = -4$ $y=-7$



3. This graph represents a linear relation.

a) Determine the value of x for each value of y.

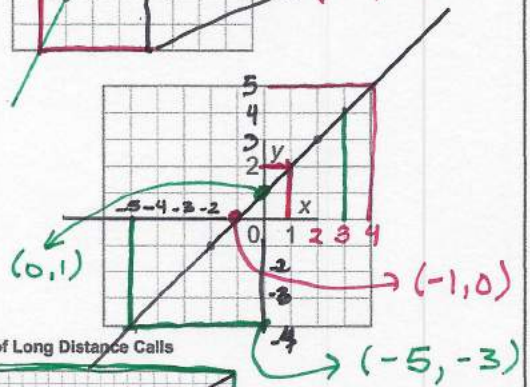
i) $y = 2$ $x=1$ ii) $y = 0$ $x=-1$

iii) $y = 5$ $x=4$

b) Determine the value of y for each value of x.

i) $x = 0$ $y=1$ ii) $x = 3$ $y=4$

iii) $x = -5$ $y=-3$



4. The graph shows how the cost of a long distance call changes with the time for the call.

a) Estimate the cost of a 7-min call. ≈ 0.55

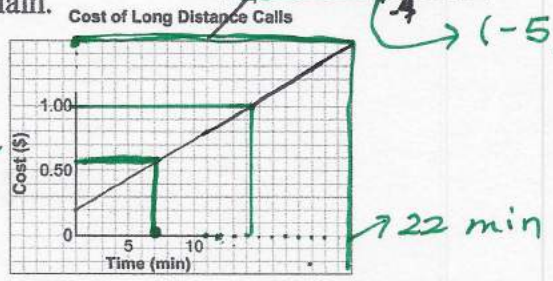
Is this interpolation or extrapolation? Explain.

b) The cost of a call was \$1.00.

Estimate the time for the call. ≈ 14 min

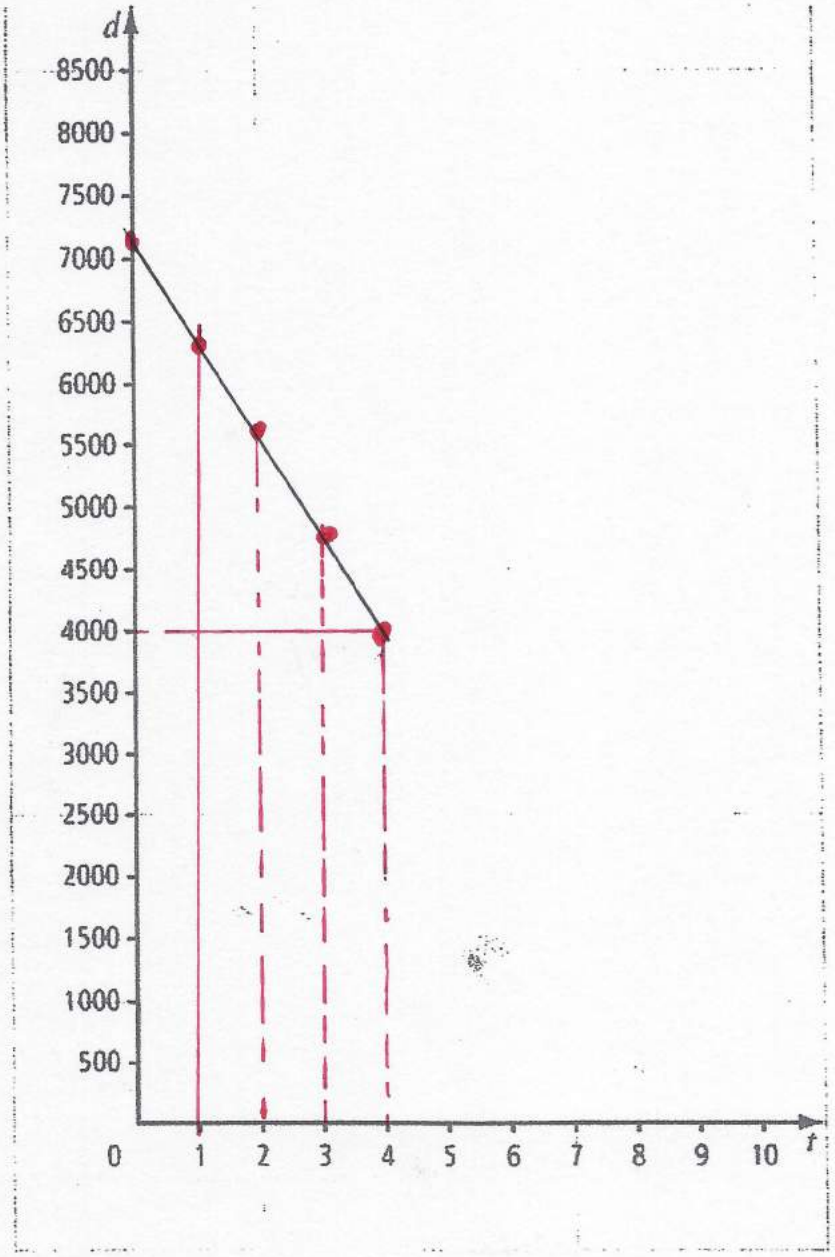
c) The cost of a call was \$1.50.

Estimate the time for the call. ≈ 22 min



Name: _____ ID: A

13. A jet flies from Toronto to Rome. Its flight can be modelled by the linear equation $d = 7200 - 800t$, where d is the distance, in kilometres, from Rome and t is the time, in hours.
a) Graph the linear relation.
b) How long does it take to fly 4000 km?



Make a Table

$d = 7200 - 800t$

t(x)	d(y)	
0	7200	) -800
1	6400	
2	5600	
3	4800	
4	4000	

GRAPHING

1. a) Create a table of values for the relation $y = 1\frac{1}{2}x + 3$, then graph the relation.

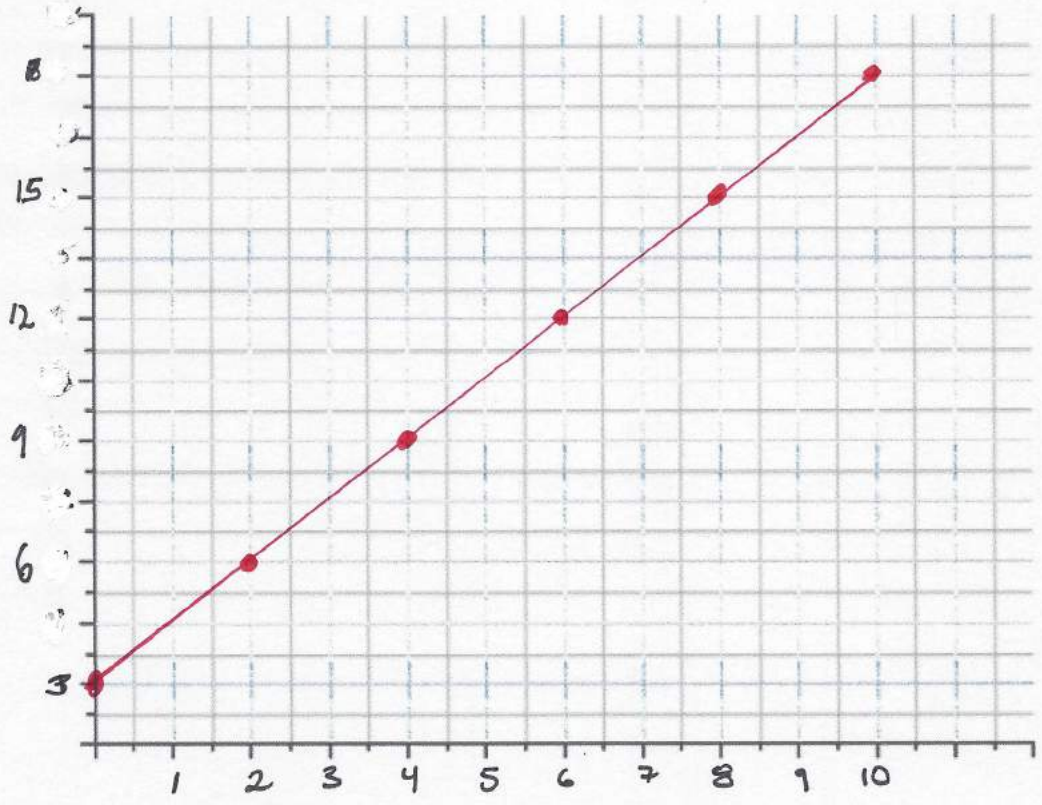
x	y
0	3
2	6
4	9
6	12
8	15
10	18

+3

$y = 1.5x + 3 = 1.5(0) + 3 = 3$

$y = 1.5(2) + 3 = 6$

the pattern is +3



b) Is the relation linear? How do you know? x has a pattern, y has a pattern

c) Extrapolate the value of y when $x = 33$.

$y = 1.5x + 3$

$= 1.5(33) + 3 =$ 52.5

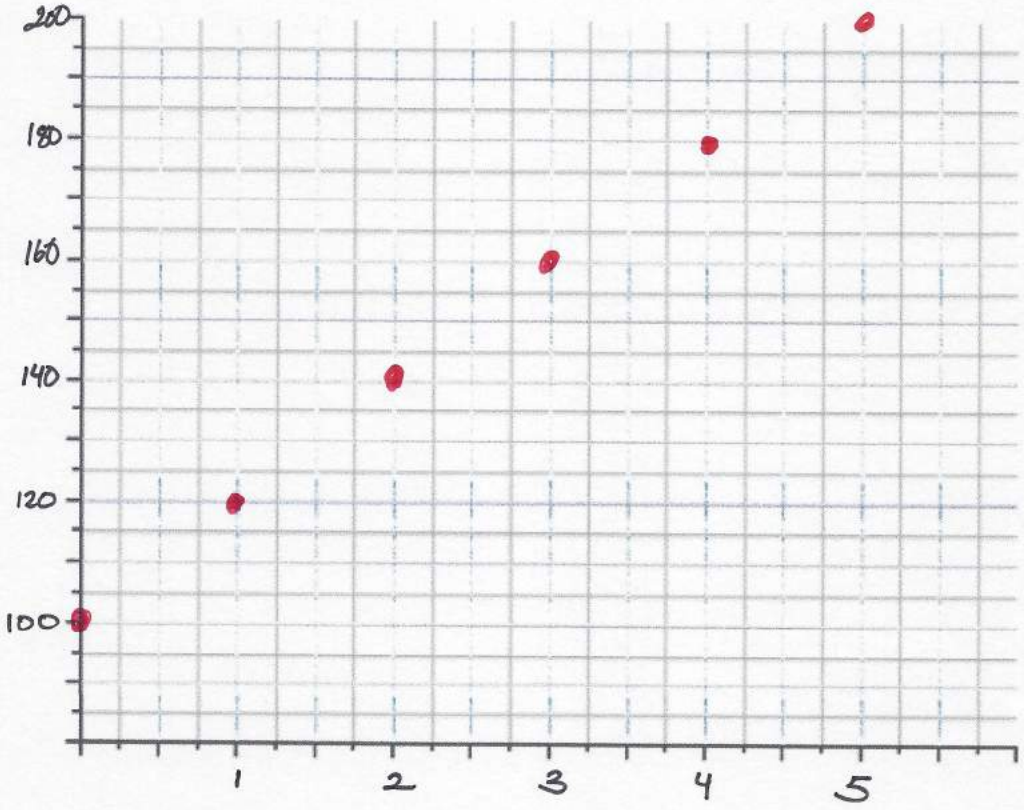
2. Sam has $100 in his savings account. Each week he adds $20.

a. Write an equation that relates the amount of money in his account, A dollars, after t weeks.

$A = 100 + 20t$

b. Create a table of values for the relation, then graph it. Will you join the points on this graph? Explain. No, data is finite

t	0	1	2	3	4	5
A	100	120	140	160	180	200



c. At what point will Sam have $250.00 in his account?

$250 = 100 + 20t$

$20t = 150$ $t = 7.5$ Weeks