Grade 9 Mathematics
Unit 2: Powers and Exponent Rules

Sec 2.1 What is a Power

$\underbrace{2^5}$ 2 is the BASE
5 is the EXPONENT

The entire 2^5 is called a POWER.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2$ written as repeated multiplication.

$2^5 = 32$ written in standard form.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

Power | Repeated Multiplication | Standard Form

To evaluate a power means to find the answer in standard form.

Are the base and the exponent interchangeable? In other words, does $2^5 = 5^2$?

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ $\quad$ $5^2 = 5 \times 5 = 25$

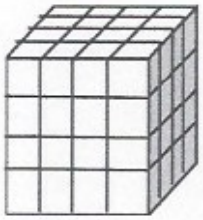
- No, the base and exponent cannot be switched and still be equal.

CHALLENGE!!!! Can you think of one example where the base and exponent can be switched, and the answers are still equal?

4^2 When you have an exponent of 2, it's called a **squared number.**

$4^2 = 4 \times 4 = 16$

4^3 When you have an exponent of 3, it's called a **cubed number.**



$4^3 = 4 \times 4 \times 4$
$= 64$

The Importance of Brackets

$(-3)^2$ The brackets tell us that the base is −3.

- $(-3)^2 = (-3) \times (-3) = +9$

When there is an EVEN NUMBER of negatives then the product is POSITIVE.

- $(-3)^3 = (-3) \times (-3) \times (-3) = -9$

When there is an ODD NUMBER of negatives then the product is NEGATIVE.

-3^2 There are no brackets so the base is 3. The negative applies to the whole expression.

- $-3^2 = -(3 \times 3) = -9$

Question.

1. Identify the base and evaluate each power.

a). $(-5)^4 = 625$ Base: (-5)

b). $-5^4 = -625$ Base: 5

C). $-(-5)^4 = -625$ Base: (-5)

d). $(-5)^3 = -125$ Base: (-5)

e). $-5^3 = -125$ Base: 5

f). $-(-5)^3 = -(-125) = +125$ Base: (-5)

Sec 2.2 Powers of Ten and the Exponent Zero

Investigation

Power	Repeated Multiplication	Standard Form
3^5	$3 \times 3 \times 3 \times 3 \times 3$	243
3^4	$3 \times 3 \times 3 \times 3$	81
3^3	$3 \times 3 \times 3$	27
3^2	3×3	9
3^1	3	3

Look for the patterns in the columns.

The exponent decreases by 1 each time.

Each time the exponent decreases, standard form in divided by 3.

This pattern suggests that $3^0 =$ 1.

A power with exponent 0 is equal to 1.

1a). Complete the table below.

Power	Repeated Multiplication	Standard Form
5^4	$5 \times 5 \times 5 \times 5$	625
5^3	$5 \times 5 \times 5$	125
5^2	5×5	25
5^1	5	5

b). What is the value of 5^1 ? 5

c). What is the value of 5^0 ? 1

Zero Exponent Rule:

Any base (excluding zero) with the exponent zero is one.

$a^0 = 1$

where $a \neq 0$

Examples: Remember, any **base** with the **exponent zero** is **one.**

1. Identify the base, then evaluate the answer.

a). $5^0 = 1$ b). $10^0 = 1$ c). $(-5)^0 = 1$ d). $-10^0 = -1$

Base = 5 Base = 10 Base = (-5) Base = 10

2. Evaluate the following powers. Remember the order of operations!

a). $3 + 2^0 = 3 + 1 = 4$ b). $3^0 + 2^0 = 1 + 1 = 2$ c). $(3 + 2)^0 = (5)^0 = 1$

d). $-3^0 + 2 = -1 + 2 = 1$ e). $-3^0 + (-2)^0 = -1 + 1 = 0$ f). $-(3 + 2)^0 \quad -(5)^0 = -1$

Writing Powers of Ten

Complete the missing values.

Power	Repeated Multiplication	Standard Form	Words
10^3	$10 \times 10 \times 10$	1000	1 thousand
10^5	$10 \times 10 \times 10 \times 10 \times 10$	100 000	hundred thousand
10^6	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	1000000	1 million
10^2	10×10	100	hundred
10^1	10	10	ten
10^0	1	1	one

ANSWERS

Power	Repeated Multiplication	Standard Form	Words
10^3	$10 \times 10 \times 10$	1000	1 thousand
10^5	$10 \times 10 \times 10 \times 10 \times 10$	100 000	hundred thousand
10^6	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	1 000 000	1 million
10^2	10×10	100	1 hundred
10^1	10	10	ten
10^0	———	1	one

Section 2.3 Order of Operations

Review the basics

Adding Integers

(+5) + (+2) = +7
(-6) + (-4) = -10
(-8) + (+2) = -6
(+9) + (-3) = +6

Subtracting Integers

(+7) - (+3) = (+4)
(-6) - (-3) = (-6) + (+3) = -3
(-2) - (+9) = (-2) + (-9) = -11
(+3) - (-6) = (+3) + (+6) = +9

When subtracting remember to "Add the Opposite"

Multiplying Integers

(+2)(+3) = +6
(-4)(-5) = +20
(+3)(-5) = -15
(-2)(+7) = -14

When multiplying or dividing:

$+ \times + = +$
$- \times - = +$
} same signs is positive

$- \times + = -$
$+ \times - = -$
} different signs is negative

Dividing Integers

(+10) ÷ (+2) = +5
(-45) ÷ (-5) = +9
(-121) ÷ (+11) = -11
(+64) ÷ (-8) = -8

Order of Operations

B - do operations inside brackets first

E - exponents

D, M } multiply or divide, in order, from left to right, whichever comes first

A, S } add or subtract, in order, from left to right, whichever comes first

Examples

A). $2^3 + 1$

$(2)(2)(2) + 1$

$8 + 1$

9

B). $8 - 3^2$

$8 - (3)(3)$

$8 - 9$

$8 + -9$

-1

C). $(3-1)^3$

$(2)^3$

8

D). $[2 \times (-2)^3]^2$

$[2 \times (-2)(-2)(-2)]^2$

$[2 \times (-8)]^2$

$[-16]^2$

$(-16)(-16)$

256

E).

$(7^2 + 5^0) \div (-5)^1$

$[(7)(7) + 1] \div (-5)^1$

$[49 + 1] \div (-5)^1$

$50 \div -5$

-10

F). This student got the correct answer, but did not earn full marks. Find and explain the mistake the student made.

$$-(24-3\times 4^2)^0 \div (-2)^3$$
$$-(24-12^2)^0 \div (-8)$$
$$-(24-144)^0 \div (-8)$$
$$-(-120)^0 \div (-8)$$
$$-1 \div (-8)$$
$$\frac{1}{8}$$

The mistake occurred at 4^2. $4^2 = 16$ should have been done before 3×4. Or the student could have realized that the entire bracket has the exponent zero, so it's 1.

$$-(24-3\times 4^2)^0 \div (-2)^3$$
$$-(1) \div (-2)^3$$
$$-1 \div (-8)$$
$$\frac{1}{8}$$

Section 2.4 Exponent Laws I

Product of Powers Investigation

1: Complete the table below.

Product of Powers	Repeated Multiplication	Power Form
$10^2 \times 10^3$	$(10\times10)\times(10\times10\times10)$	10^5
$10^3 \times 10^4$	$(10\times10\times10)\times(10\times10\times10\times10)$	10^7
$5^4 \times 5^5$	$(5\times5\times5\times5)\times(5\times5\times5\times5\times5)$	5^9
$2^3 \times 2^1$	$(2\times2\times2)\times2$	2^4
$3^2 \times 3^5$	$(3\times3\times3)\times(3\times3\times3\times3\times3)$	3^7
$4^3 \times 4^2$	$(4\times4\times4)\times(4\times4)$	4^5

2: Create 5 more examples of your own.

Product of Powers	Repeated Multiplication	Power Form

3: State a rule for multiplying any two powers with the same base.

To Multiply powers with the same base, you ADD the exponents.

4: Can you use your rule to multiply $2^3 \times 3^2$? Explain why or why not?

No. these powers have different bases.

Quotients of Powers Investigation

1: Complete the table below.

Quotient of Powers	Repeated Multiplication	Power Form
$10^5 \div 10^3$	$\frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10}$	10^2
$10^8 \div 10^5$	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$	10^3
$5^{10} \div 5^4$	$\frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5}$	5^6
$9^8 \div 9^3$	$\frac{9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9}{9 \cdot 9 \cdot 9}$	9^5
$7^5 \div 7^4$	$\frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7 \cdot 7}$	7^1
$4^7 \div 4^4$	$\frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4}$	4^3

2: Create 5 more examples of your own. Make sure you put the larger exponent first!

Quotient of Powers	Repeated Multiplication	Power Form

3: State a rule for dividing two powers with the same base.

When dividing powers with the same base, the exponents are subtracted.

4: Can you use your rule to divide $5^2 \div 2^3$? Explain why or why not?

No. These powers have different bases.

Summary Notes

Exponent Law for a Product of Powers	$a^m \times a^n = a^{m+n}$
To multiply powers with the same base, (excluding a base of zero), keep the base and add the exponents.	where $a \neq 0$ and m and n are whole numbers

1. Write as a single power, then evaluate.

a). $4^3 \times 4^4$ $4^{3+4} = 4^7$ 16,384

b). $7^5 \times 7^{-5}$ $7^{5+(-5)} = 7^0$ 1

c). $(-3)^2 \times (-3)^4$ $(-3)^{2+4} = (-3)^6$ 729

2. Write as a single power.

a). $9^5 \times 9 = 9^6$ 531,441

b). $8^{-11} \times 8^{13} = 8^2$ 64

c). $3.8^4 \times 3.8^2 = 3.8^6$ 3,010.9

d). $\left(\frac{1}{4}\right)^{12} \times \left(\frac{1}{4}\right)^8 = \left(\frac{1}{4}\right)^{20}$ 1 Big number

e). $5^2 \times 5 \times 5^3 = 5^{2+1+3} = 5^6$ 15,625

Exponent Law for a Quotient of Powers	$a^m \div a^n = a^{m-n}$
To divide powers with the same base, (excluding a base of zero), keep the base and subtract the exponents.	where $a \neq 0$ and m and n are whole numbers and $m \geq n$.

3. Write as a single power, then evaluate.

a). $2^5 \div 2^2 = 2^{5-2} = 2^3$ 8

b). $\frac{(-6)^8}{(-6)^6} = (-6)^2$ 36

c). $\frac{3^4}{3^4} = 3^{4-4} = 3^0 = 1$

4. Write as a single power.

a). $12^6 \div 12$

$= 12^{6-1}$

$= 12^5$

$= 248,832$

b). $\frac{8^3}{8^{-2}}$

$= 8^{3-(-2)}$

$= 8^5$

$= 32,768$

c). $(1.4)^6 \div (1.4)^2$

$= (1.4)^{6-2}$

$= (1.4)^4$

$= 3.8416$

d). $\frac{x^7}{x^5} = x^2$

e). $\frac{5^7}{5^3} = 5^{7-3} = 5^4 = 625$

Note: "Evaluate" means to find the answer in "standard form"

Example: Evaluate 4^3 $= 4 \times 4 \times 4 = 64$

Evaluate: $2^3 \times 2^2$

$= 2^{3+2}$

$= 2^5$

$= 32$

"Express as a single power" means leave your answer in "exponent form"

$$\frac{5^8}{5^2} = 5^{8-2} = 5^6$$

Examples:

1. Express as a single power

a) $5^2 \times 5^4 \times 5$

$= 5^{2+4+1}$

$= 5^7 = 78,125$

b) $6^{-6} \times 6^2$

$= 6^{-6+2}$

$= 6^{-4}$

c) $(-6)^7 \div (-6)^6$

$= (-6)^{7-6}$

$= (-6)^1$

d) $10^8 \div 10^2$

$= 10^{8-2}$

$= 10^6$

*** Often you will have problems where you will have to apply more than one exponent law.

e) $8^{12} \div 8^7 \times 8^2$

$= 8^{12-7} \times 8^2$

$= 8^5 \times 8^2$

$= 8^{5+2}$

$= 8^7$

f) $\frac{2^3 \times 2^5}{2^2} = \frac{2^{3+5}}{2^2} = \frac{2^8}{2^2} = 2^6$

Evaluate:

g) $\dfrac{(-4)^{10}}{(-4)^3 \times (-4)^3} = \dfrac{(-4)^{10}}{(-4)^{3+3}} = \dfrac{(-4)^{10}}{(-4)^6} = (-4)^{10-6} = (-4)^4 = 256$

h) $6^2 + 6^3 \times 6^2$
$= 6^2 + 6^{3+2}$
$= 6^2 + 6^5$
$= 36 + 7776$
$= 7812$

i) $(-10)^4 [(-10)^6 \div (-10)^4] - 10^7$

$= (-10)^4 [(-10)^{6-4}] - 10^7$
$= (-10)^4 (-10)^2 - 10^7$
$= (-10)^{4+2} - 10^7$
$= (-10)^6 - 10^7$
$= 1\,000\,000 - 10\,000\,000$
$= -9\,000\,000$

Power of a Power Investigation 1

1: Complete the table below.

Power of a Power	Repeated Multiplication	Product of Factors	Power Form
$(2^4)^3$	$2^4 \times 2^4 \times 2^4$	$(2\times2\times2\times2)\times(2\times2\times2\times2)\times(2\times2\times2\times2)$	2^{12}
$(3^2)^4$	$3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2$	$(3\cdot3)(3\cdot3)(3\cdot3)(3\cdot3)$	3^8
$(4^2)^3$	$4^2 \cdot 4^2 \cdot 4^2$	$(4\cdot4)(4\cdot4)(4\cdot4)$	4^6
$(5^3)^3$	$5^3 \cdot 5^3 \cdot 5^3$	$(5\cdot5\cdot5)(5\cdot5\cdot5)(5\cdot5\cdot5)$	5^9
$[(-4)^3]^2$	$(-4)^3 \cdot (-4)^3$	$(-4)(-4)(-4) \times (-4)(-4)(-4)$	$(-4)^6$
$[(-5)^3]^5$	$(-5)^3 \cdot (-5)^3 \cdot (-5)^3 \cdot (-5)^3 \cdot (-5)^3$	$(-5)(-5)(-5) \times (-5)(-5)(-5) \times (-5)(-5)(-5) \times (-5)(-5)(-5) \times (-5)(-5)(-5)$	$(-5)^{15}$

2: State a rule for when you have two exponents (power of a power).

When there are 2 exponents, such exponents are multiplied.

Section 2.5

Power of a Product Investigation 2

1: Complete the table below.

Power	Repeated Multiplication	Product of Factors	Product of Powers
$(2\times5)^3$	$(2\times5)\times(2\times5)\times(2\times5)$	$2\times2\times2\times5\times5\times5$	$2^3\times5^3$
$(3\times4)^2$	$(3\times4)\times(3\times4)$	$3\times3\times4\times4$	$3^2\times4^2$
$(4\times2)^5$	$(4\times2)\cdot(4\times2)\cdot(4\times2)\cdot(4\times2)\cdot(4\times2)$	$4\times4\times4\times4\times4\times2\times2\times2\times2\times2$	$4^5\times2^5$
$(5\times3)^4$	$(5\times3)(5\times3)(5\times3)(5\times3)$	$5\times5\times5\times5\times3\times3\times3\times3$	$5^4\times3^4$
$(5\times6)^2$	$(5\times6)(5\times6)$	$5\times5\times6\times6$	$5^2\times6^2$
$[7\times(-2)]^3$	$[7\times(-2)][7\times(-2)][7\times(-2)]$	$7\times7\times7\times(-2)\times(-2)\times(-2)$	$7^3\times(-2)^3$

2: State a rule for when you have a power of a product.

For a power of a product, the outside exponent multiplies every exponent inside the bracket.

Section 2.5

Power of a Quotient Investigation 3

1: Complete the table below.

Power	Repeated Multiplication	Product of Factors	Product of Quotients
$\left(\frac{5}{6}\right)^3$	$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$	$\frac{5 \times 5 \times 5}{6 \times 6 \times 6}$	$\frac{5^3}{6^3}$
$\left(\frac{2}{3}\right)^4$	$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$	$\frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}$	$\frac{2^4}{3^4}$
$\left(\frac{1}{5}\right)^5$	$\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$	$\frac{1 \times 1 \times 1 \times 1 \times 1}{5 \times 5 \times 5 \times 5 \times 5}$	$\frac{1^5}{5^5}$
$\left(\frac{3}{10}\right)^2$	$\frac{3}{10} \times \frac{3}{10}$	$\frac{3 \times 3}{10 \times 10}$	$\frac{3^2}{10^2}$
$\left(\frac{-4}{7}\right)^3$	$\left(\frac{-4}{7}\right)\left(\frac{-4}{7}\right)\left(\frac{-4}{7}\right)$	$\frac{(-4)(-4)(-4)}{7 \times 7 \times 7}$	$\frac{(-4)^3}{7^3}$
$\left(\frac{-4}{-5}\right)^6$	$\left(\frac{-4}{-5}\right)\left(\frac{-4}{-5}\right)\left(\frac{-4}{-5}\right)\left(\frac{-4}{-5}\right)\left(\frac{-4}{-5}\right)\left(\frac{-4}{-5}\right)$	$\frac{(-4)(-4)(-4)(-4)(-4)(-4)}{(-5)(-5)(-5)(-5)(-5)(-5)}$	$\frac{(-4)^6}{(-4)^5}$

2: State a rule for when you have a power of a quotient.

In a power of a quotient, the exponent outside the bracket multiplies the exponents of the powers involved in the quotient

Exponent Law for a Power of a Power

When you have a power to a power, the base stays the same and multiply the exponents.

$$(a^m)^n = a^{m \times n}$$

where $a \neq 0$ and m and n are whole numbers

1. Write as a power.

a). $(3^2)^4$

$= 3^{2 \times 4}$

$= 3^8$

b). $[(-7)^3]^2$

$= (-7)^{3 \times 2}$

$= (-7)^6$

c). $-(2^2)^4$

-2^8

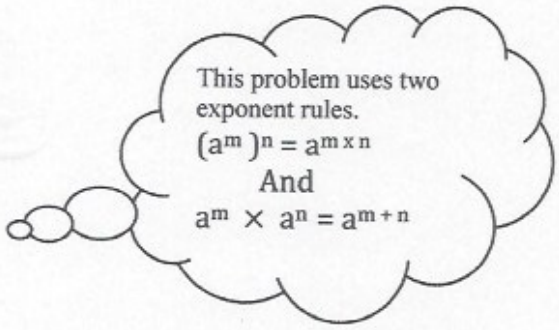
d). $(3^0)^2$

3^0

e). $(42^3)^2 \times (42^4)^4$

$= 42^6 \times 42^{16}$

$= 42^{22}$



2. Simplify first, then evaluate.

a). $(2^3)^2 \times (3^2)^2$

$= 2^{3 \times 2} \times 3^{2 \times 2}$

$= 2^6 \times 3^4$

$= 64 \times 81$

$= 5184$

b). $(-3^2)^3 \times (-3^0)^9$

$= (-3)^6 \times (-3)^0$

$= (-3)^6 \times 1$

$= (-3)^6$

$= 729$

Exponent Law for a Power of a Product

When you have a power of a product, the exponent outside of the bracket is applied to the exponents on each of the factors on the inside of the brackets.

$$(ab)^m = a^m b^m$$

where $a \neq 0$ and $b \neq 0$ and m is a whole number

1. Evaluate each question two ways. Use power of a product and BEDMAS.

a). $[(-7) \times 5]^2$

Method 1:

$= (-7)^2 \times 5^2$

$= 49 \times 25$

$= 1225$

Method 2:

$[(-7) \times 5]^2$

$= [-35]^2$

$= (-35) \times (-35)$

$= 1225$

b). $-(3 \times 2)^2$

Method 1:

$= -(3^2 \times 2^2)$

$= -(9 \times 4)$

$= -36$

Method 2:

$-(3 \times 2)^2$

$= -(6)^2$

$= -36$

2. Evaluate, using any method of your choice.

a). $(3 \times 4)^3$

$= (12)^3$

$= 1728$

OR

$(3^3 \times 4^3)$

$= 27 \times 64$

$= 1728$

b). $[(-2)^2 \times (-2)^1]^3$

$= [(-2)^3]^3$

$= (-2)^9$

$= -512$

OR

$= [(-2)^6 \times (-2)^3]$

$= [64 \times (-8)]$

$= -512$

Exponent Law for a Power of a Quotient

When you have a power of a quotient, the exponent outside of the bracket is applied to the exponents on the numerator and denominator of the fraction inside of the brackets.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

where $a \neq 0$ and $b \neq 0$ and m is a whole number

1. Evaluate each question two ways. Use power of a quotient and BEDMAS.

a). $[(-24) \div 6]^4$

Method 1:

$= (-24)^4 \div 6^4$

$= 331776 \div 1296$

$= 256$

Method 2:

$[(-24) \div 6]^4$

$= [-4]^4$

$= 256$

b). $\left(\frac{52}{13}\right)^3$

Method 1:

$(52)^3 \div (13)^3$

$140608 \div 2197$

$= 64$

Method 2:

$\left(\frac{52}{13}\right)^3$

$= (4)^3$

$= 64$