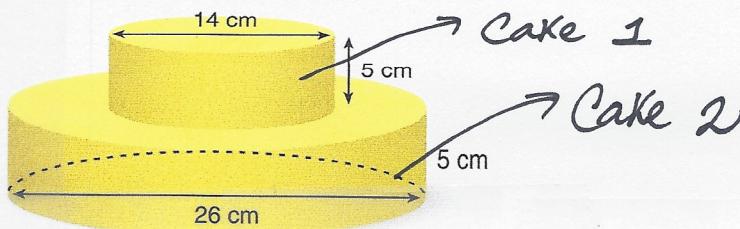


Example 2**Determining the Surface Area of a Composite Object Made from Two Cylinders**

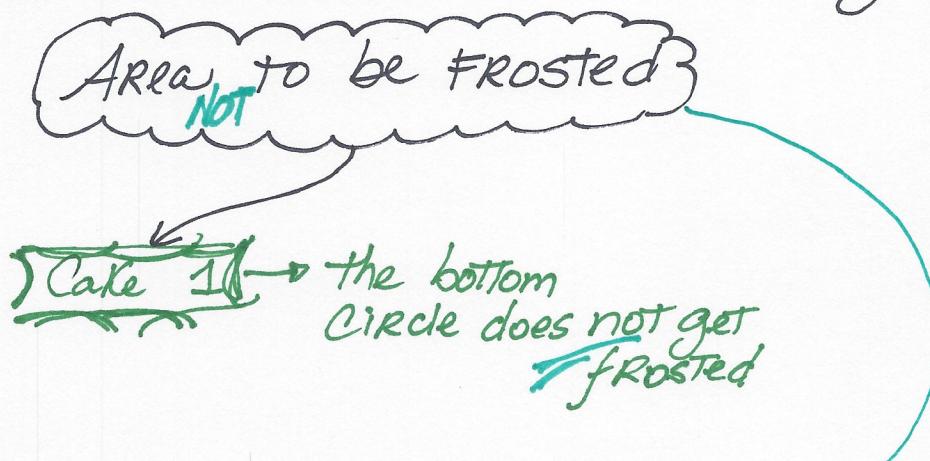
Two round cakes have diameters of 14 cm and 26 cm, and are 5 cm tall.

They are arranged as shown. The cakes are covered in frosting. What is the area of frosting?



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your textbook!

Let's think about what is being asked:



Everything else gets frosted:

Cake 1
- top face
- curved side face

Cake 2

- top face MINUS an area equal TO the bottom of Cake 1
- curved side face

-
- An area on the top surface EQUAL TO the bottom of Cake 1 (Not Frosted)
 - The bottom surface does not get frosted

HERE ARE A COUPLE OF WAYS
TO SOLVE THIS

METHOD 1

Fig 1 : $d = 14 \text{ cm}$, $r = 7 \text{ cm}$
 $h = 5 \text{ cm}$

Fig 2 : $d = 26 \text{ cm}$, $r = 13 \text{ cm}$
 $h = 5 \text{ cm}$

Figure 1 → the bottom circle will
 NOT be frosted : So :

$$S_A \text{ Fig 1} = \pi r^2 + (2\pi r \times h)$$

without
bottom
circle

$$\begin{aligned} S_A \text{ fig 1} &= \pi (7 \text{ cm})^2 + (2 \times 7 \times 3.14 \times 5) \\ &= \pi 49 \text{ cm}^2 + \pi 70 \text{ cm}^2 \\ &= 153.93 \text{ cm}^2 + 219.91 \text{ cm}^2 \\ &= 373.84 \text{ cm}^2 \end{aligned}$$

In figure 2, the bottom circle
 AND an area equal to that of the
 bottom circle of fig. 1 are NOT
 to be frosted. So

$$S_A \text{ to be frosted in fig. 2} = \pi r^2 + (2\pi r \times h)$$

$$\begin{aligned} (\text{without bottom circle}) &= \pi (13)^2 + (2\pi 13 \times 5) \\ &= \pi \times 169 \text{ cm}^2 + 26\pi \times 5 \\ &= 530.92 \text{ cm}^2 + 408.40 \text{ cm}^2 \\ &= 939.32 \text{ cm}^2 \end{aligned}$$

But, an area equal to the area of fig. 1's bottom circle has to be subtracted because its area does not get frosted

So

Area

$$1 \text{ bottom circle} = \pi \times r^2$$

$$= \pi \times (7\text{cm})^2$$

$$\text{of Fig. 1} = \pi \times 49 = 153.93 \text{ cm}^2$$

Note :

In this method, we already took out one of the two surfaces that make up the overlap. therefore, we don't have to get the overlap.

Total Area to be frosted :

$$(939.32 \text{ cm}^2) + (373.84 \text{ cm}^2) - (153.93 \text{ cm}^2)$$

$$= 1159.23 \text{ cm}^2$$

Now, let's try a slightly different approach:

Method 2

- First • Get the area of fig. 1
- Second • get the area of fig. 2 (minus the bottom circle since it is not to be frosted)
- then • Add the areas together
- And • Subtract the overlap (which is equal to the area of the circle in fig 1 + 2)

Fig 1

$$\begin{aligned}
 SA_{\text{fig 1}} &= 2\pi r^2 + (2\pi r \times h) \\
 &= 2\pi(7\text{cm})^2 + (2 \times \pi \times 7\text{cm} \times 5\text{cm}) \\
 &= 2\pi \times 49\text{ cm}^2 + 2\pi \times 35\text{ cm}^2 \\
 &= 98\pi\text{ cm}^2 + 70\pi\text{ cm}^2 \\
 &= 168\pi\text{ cm}^2 \\
 &= \boxed{527.78\text{ cm}^2}
 \end{aligned}$$

$$\begin{aligned}
 SA_{\text{fig 2}} &= \pi r^2 + (2\pi r \times h) \\
 (\text{less one circle}) &= \pi(13)^2\text{ cm} + (2\pi \times 13\text{cm} \times 5\text{cm}) \\
 &= 169\pi\text{ cm}^2 + 130\pi\text{ cm}^2 \\
 &= 299\pi\text{ cm}^2 \\
 &= \boxed{-939.33\text{ cm}^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{Overlap} &= \text{Area of bottom circle} \times 2 \\
 &= \pi r^2 \times 2 \\
 &= \pi (7\text{cm})^2 \times 2 \\
 &= 98\pi\text{ cm}^2 = 307.87\text{ cm}^2
 \end{aligned}$$

So,

$$\begin{aligned} \text{Area to be frosted} &= S_A \text{ Figure 1} + S_A \text{ Figure 2} - \text{overlap} \\ &= 527.78 \text{ cm}^2 + 939.33 \text{ cm}^2 - 307.87 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \rightarrow \text{Total} &= 1467.11 \text{ cm}^2 - 307.87 \text{ cm}^2 \\ &= \underbrace{1159.24} \end{aligned}$$

See? Same Result,

Different Approach!