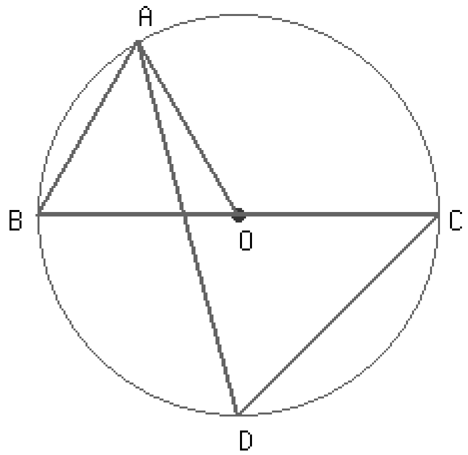
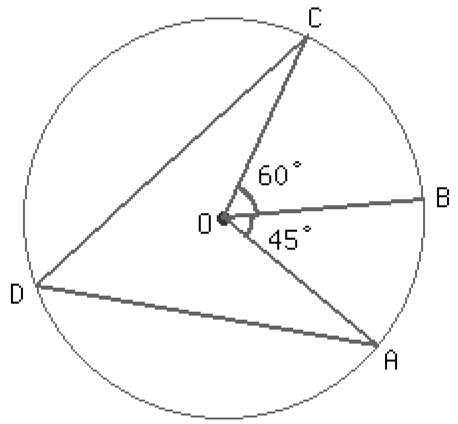
## Answer the questions

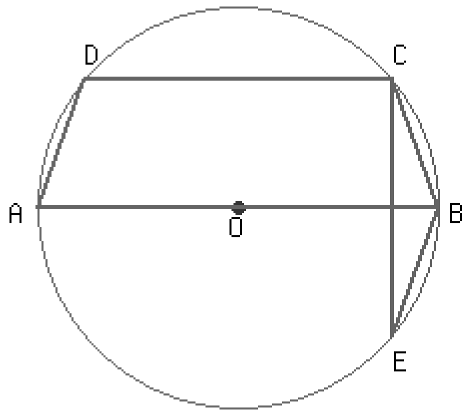
**(1)** If BC is a diameter of the circle and ∠BAO = 60°. Then find the value of ∠ADC.



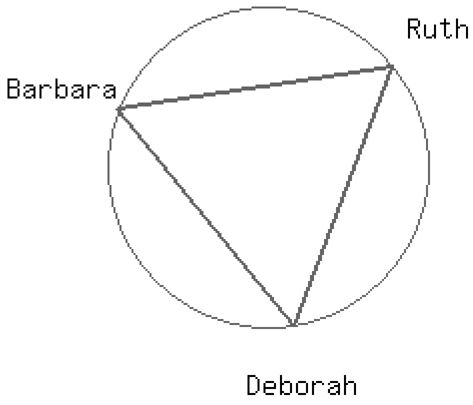

**(2)** If O is center of the circle, find angle ∠ADC.



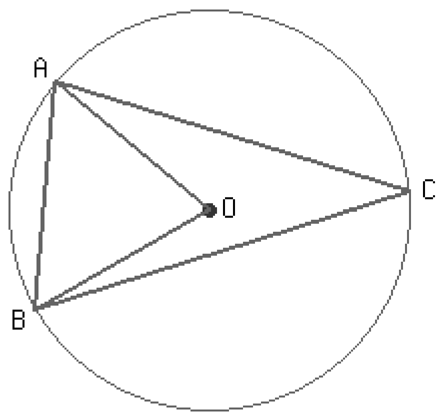

**(3)** If ∠ADC = 110° and chord BC = chord BE. Find ∠CBE.



**(4)** There is a circular park of radius 16 meters. Three friends Ruth, Barbara and Deborah are sitting at equal distance on its boundary each having a toy telephone (connected using strings) in their hands to talk each other. Find the length of the string between a pair of the telephones.

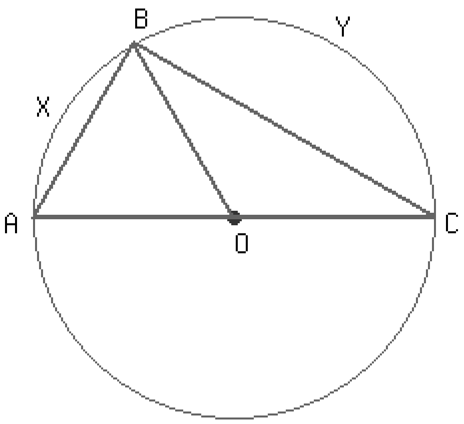



**(5)** If O is center of the circle and ∠OAB = 55° , the measure of ∠ACB is:

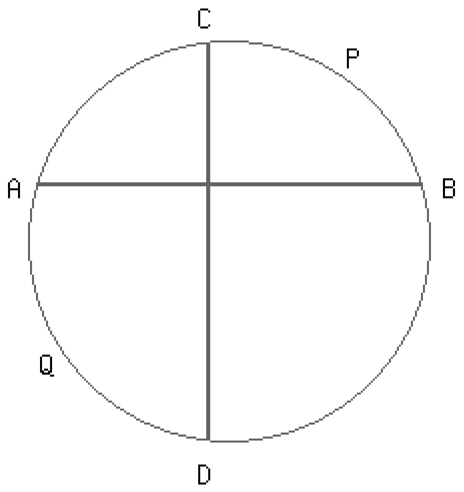



**(6)** Two chords AB and AC of a circle subtends angles equal to 110° and 80°, respectively at the centre. Find ∠BAC, if AB and AC lie on the opposite sides of the centre.

**(7)** AC is a diameter of the circle and ***arc AXB*** = $\frac{1}{2}$ *arc BYC*. Find ∠BOC.

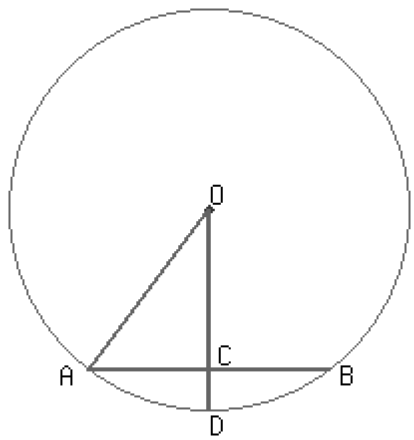


**(8)** The chords AB and CD of a circle are perpendicular to each other. If radius of the circle is 28 cm and length of the arc BPC is 40 cm, find the length of arc AQD. (Assume $\pi = \frac{22}{7}$ )




**(9)** A chord of a circle is equal to its radius. Find the angle subtended by this chord at a point in the major segment.

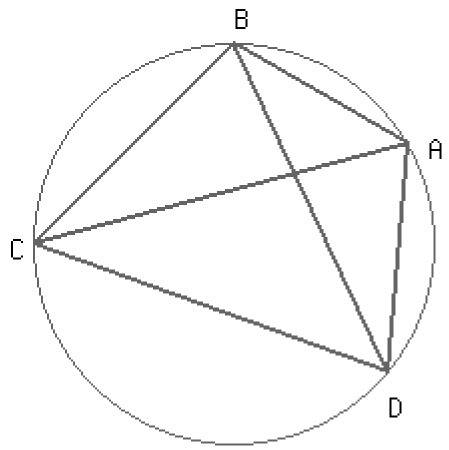
**(10)** If CD = 1 cm, OA = 5 cm and OD is perpendicular to AB, then AB is:




**(11)** The Ferris Wheel at the school fair has radius of 8 metres. It revolves at the rate of one revolution per 2 minutes. How many seconds does it take a rider to travel from the bottom of the wheel to a point 4 vertical metres above the bottom?

**(12)** AB and AC are two chords of a circle such that AB = 2AC. If distances of AB and AC from the centre are 2 cm and 4 cm respectively, find the area of circle. (Assume $\pi = 3$)

**(13)** If ∠BAD = 115° and ∠ABD = 35°, find angle ∠ACB.




## Check True/False

**(14)** If two chords AB and CD of a circle are at a distance of 16 cm from the centre, then AB = CD.

○ True ○ False

**(15)** Two congruent circles with centres O and O′ intersect at two points P and Q. Then, ∠POQ = ∠PO'Q.

○ True ○ False

# Answers

**(1)** 60°

**Step 1**

As,OA and OB are the radius of the circle, OA = OB. This means ΔAOB is an isosceles triangle. So,
∠ABO = ∠BAO = 60°.

**Step 2**

Also, ∠ABO = ∠ABC
Considering the chord AC, ∠ABC and ∠ADC are the angles subtended by the chord AC in the same segment of the circle.
We know that the angle subtended by a chord in the same segment of a circle are equal. So,
∠ABC = ∠ADC

**Step 3**

Therefore, ∠ADC = ∠ABC = 60°.

**(2)** 52.5°

We see in the image that AC is a chord of the circle
The angle subtended by the chord to the center is twice the angle subtended to a point on the circumference (on the same side as the center)
The total angle subtended to the center by AC = 45 + 60 = 105
Therefore the angle subtended to D which lies in the circumference = 105 ÷ 2 = 52.5°

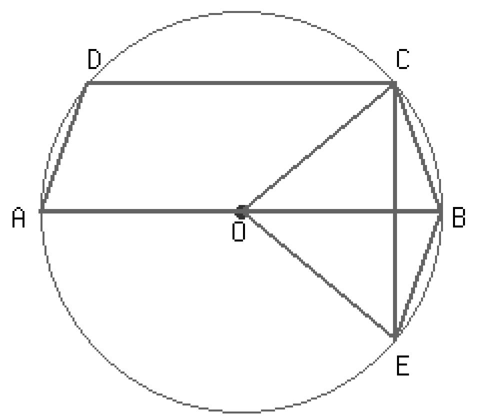
**(3)** 140°

**Step 1**

ABCD is a cyclic quadrilateral since all the four points A, B, C and D lie on the circumference of a circle.
We know, the opposite angles of a cyclic quadrilateral add up to 180°. So,
$\angle ADC + \angle CBA = 180°$
$\Rightarrow \angle CBA = 180° - \angle ADC$
$\Rightarrow \angle CBA = 180° - 110°$
$\Rightarrow \angle CBA = 70°$

**Step 2**

We know that chord BC = chord BE.
Join the points C and E to the centre of the circle.




Consider ΔCOE and ΔBOE,
BO = BO (common)
BC = BE (given)
OC = OE (radius of the circle)
So, ΔCOE ≅ ΔBOE by the property SSS.
Hence, $\angle OBC = \angle OBE$ by CPCT
We have, $\angle OBC = \angle OBE = \angle CBA = 70°$

**Step 3**

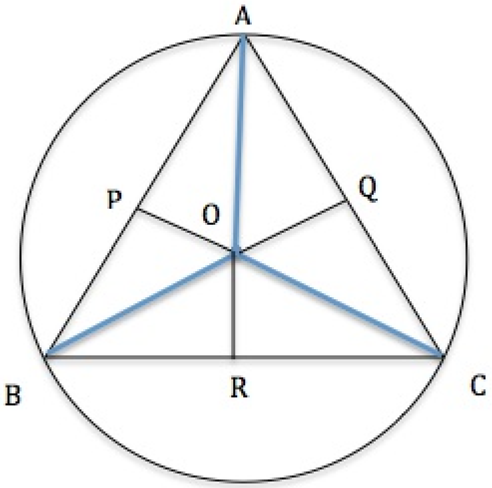
Therefore, $\angle CBE = \angle OBC + \angle OBE = 70° + 70° = 140°$.

**(4)** 16√3 m

**Step 1**

Take a look at the image below, which represents the scenario outlined in the question.




The corners of the triangle A, B and C represent the three friends Ruth, Barbara and Deborah
O is the center of the cirlce
ABC is an equilateral triangle, and we connect A, B and C to the center O.
We can see that ∠BOC = ∠COA = ∠AOB = 120° (remember, for instance ∠BAC = 60°, and the angle subtended by a chord to the center is double the angle subtended to the angle at the circumference)
We also draw perpendiculars from the center to AB, CA and BC, meeting the lines at points P, Q and R respectively
Take the triangle BOR (and remember that the same will hold true for triangles ROC, COQ, QOA, AOP and BOP)
For triangle BOR, ∠ORB = 90° , ∠RBO = 30° , BOR = 60° (since it bisects BOC), and therefore ∠RBO = 30°
So BOR is a 30°-60°-90° triangle
We know that the sides of a 30°-60°-90° triangle are in the proportion 1:$\sqrt{3}$:2
This means RO:RB:OB=1:$\sqrt{3}$:2
We know OB = radius = 16 m

**Step 2**

Therefore RB = $\frac{16\sqrt{3}}{2}$

Length of the string between A and B = 2 x RB = 16√3 m

**(5)** 35°

**Step 1**

The key point to note here is that AB is a chord of the circle,C is a point on the circumference, and O is the center.

**Step 2**

Since, OA and OB are the radius of the circle, OA = OB.
Hence, ΔOAB is an isosceles triangle.
⇒ ∠OAB = ∠OBA
In ΔAOC using angle sum property. We have,
∠OAB + ∠OBA + ∠AOB = 180°
In this case, 2 × (∠OAB) + ∠AOB = 180°
or, ∠AOB = 180° - 2 × (∠OAB)

**Step 3**

We also know that the angle subtended by a chord at the centre of a circle is double the angle subtended by the same chord at its circumference.
This means ∠AOB = 2 × ∠ACB

**Step 4**

So, we have 2 equations:
1) ∠AOB = 180° - 2 × (∠OAB)
2) ∠AOB = 2 × ∠ACB

**Step 5**

Equating both the equations and subsituting ∠OAB = 55°
We get, ∠ACB = 35°.

**(6)** 85°

**Step 1**

If we consider that the center of the circle is O, then
$\angle AOB = 110°$
and $\angle AOC = 80°$

**Step 2**

In $\Delta OAB$, we know that
$\angle BAO = \angle ABO$, (As, OB = OA and angle opposite to equal sides are equal)
and $\angle BAO + \angle ABO + \angle AOB = 180°$ (Angle sum property)
$\Rightarrow 2 \times \angle BAO + \angle AOB = 180°$

$\Rightarrow \angle BAO = \frac{1}{2} (180° - \angle AOB)$

$= \frac{1}{2} (180° - 110°)$

$= 35°$

**Step 3**

Similarly,

$\angle CAO = \frac{1}{2} (180° - \angle AOC)$

$= \frac{1}{2} (180° - 80°)$

$= 50°$

**Step 4**

Now, $\angle BAC = \angle BAO + \angle CAO$
$= 35° + 50°$
$= 85°$

**(7)** 120°

**Step 1**

The ratio of the arc length to the circumference will be the same as the angle subtended by the arc/chord to the angle in a full circle (360°).

**Step 2**

Here, we know that *arc AXB* = $\frac{1}{2}$ *arc BYC*.

This means ∠BOA = $\frac{1}{2}$ ∠BOC.

**Step 3**

Also, AC is the diameter, and angle on a straight line is 180°. So, ∠BOA + ∠BOC = 180°.

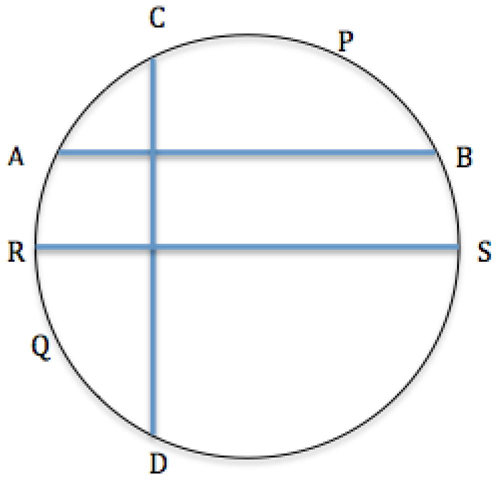
⇒ $\frac{1}{2}$ ∠BOC + ∠BOC = 180°

⇒ ∠BOC = 120°

**(8)** 48 cm

**Step 1**

Consider the figure given below:




We have drawn a diameter RS parallel to AB.

**Step 2**

Now, RS is a diameter perpendicular to CD (As, RS is parallel to AB and AB is perpendicular to CD).

We know, when a diameter is perpendicular to a chord it bisects the chord and its arc. Therefore, RS bisects arc CD.

So, Arc RC = Arc RD

Also, as AB // RS. We have, Arc AR = Arc BS

**Step 3**

Since, RS is a diameter. Arc RS cover a semi circle.

Arc RS = Arc RC + Arc CS = Arc RD + Arc CS (As, Arc RC = Arc RD)

= Arc RD + Arc CS - Arc AR + Arc AR (Adding and subtracting Arc AR)

= Arc RD + Arc CS - Arc BS + Arc AR (As, Arc AR = Arc BS)

= (Arc RD + Arc AR) + (Arc CS - Arc BS)

= Arc AQD + Arc BPC

**Step 4**

We know length of arc BPC is 40 cm, and need to find length of arc AQD.

From the previous analysis, we know that arc BPC and arc AQD cover a semi circle, so the total length of the two arcs is half the circumference.

Circumference of the circle = $2 \pi r = 2 \times \frac{22}{7} \times 28$ cm = 176 cm

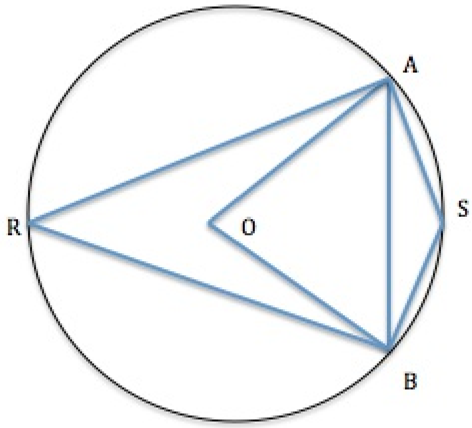
Length of arc BPC + Length of arc AQD = $\frac{1}{2} \times 176$ cm = 88 cm

Length of arc AQD = 88 cm - Length of arc BPC = 88 cm - 40 cm = 48 cm.

**(9)** 30°

**Step 1**

Look at the image below:




The chord AB has a length equal to the radius of the circle.
This means that ΔOAB is an equilateral triangle. (all the sides and angels are equal and each angle measure 60°)
So, ∠AOB = 60°.

**Step 2**

We know that the angle subtended by a chord at the center is twice the angle subtended by the chord at a point in the major segment.
Consider a point R on the major segment.
∠AOB = 2∠ARB.
Therefore, ∠ARB = 30°

**(10)** 6 cm

**Step 1**

We know, OA and OD are the radius of the circle.
Therefore, OA = OD.
Also, OD is perpendicular to chord AB i.e. OC is perpendicular to chord AB.
Therefore, $\angle OCA = 90°$.
We know that the perpendicular from the centre of a circle to a chord bisects the chord. So, the point C divides the chord AB into two equal parts, i.e., $AC = CB = \frac{AB}{2}$ .

We also know that CD = OD - OC = OA - OC
⇒ OC = OA - CD

**Step 2**

As, ΔOAC is a right-angle triangle, using pythagoras theorem. We have,
$AC^2 + OC^2 = OA^2$
From this, the relation between AB, CD and OA can be given as:

$$\left(\frac{AB}{2}\right)^2 + (OA - CD)^2 = OA^2$$

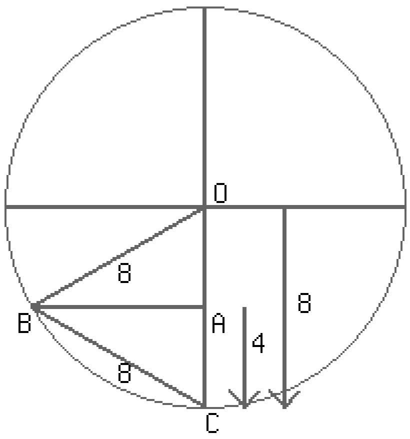
**Step 3**

Here, we know CD = 1 and OA = 5.
Substituting and simplifying in the equation we get the value of AB is 6 cm.

**(11)** 20 seconds

**Step 1**




**Step 2**

After travelling 4 vertical metres from the point of start C, its new position is B. Let us consider triangle ABO and ABC:
We have: AB is common
AC = AO = 4 m (AC = 8 - 4 = 4 m)
Angle BAO = Angle BAC (right angles)
This means triangles ABO and ABC are congruent.

**Step 3**

From step 2 we have BC = OB = 8 m. Now we have BC = OB = CO = 8 m. This means that triangle BCO is an equilateral triangle.

**Step 4**

We know that all interior angles of an equilateral triangle are equal to 60 degrees, we have Angle COB = 60.

**Step 5**

One revolution is equal to 360 degrees, which, according to the question, is completed in 2 minutes. 360º in 2 minutes
Or, 360º in 2 × 60 = 120 seconds

60º in $\frac{120}{360} \times 60$ = 20 seconds.

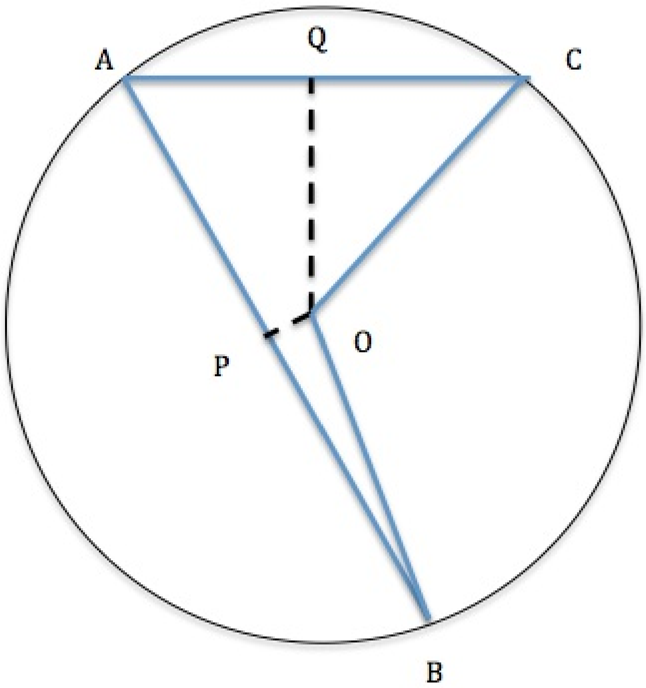
**Step 6**

It takes rider **20 seconds** to travel 4 vertical metres from the point of start.

**(12)** 60 $cm^2$

**Step 1**

Take a look at the representative image below:




We are told that AB = 2AC.
Also, if the perpendicular from O to AC meets the chord at Q, then OQ = 4 cm.
Similarly, OP = 2 cm.

**Step 2**

As the perpendicular from the centre on the chord bisects the chord, OQ bisects AC, and OP bisects AB.
From the earlier relation AB = 2AC.
Therefore, BP = 2CQ
Let us assume CQ = $x$.
Then, BP = $2x$

**Step 3**

Now consider ΔOQC,
OC = r, the radius of the circle, and OQ = 4 cm
As, the distance of a chord from the centre is always the perpendicular distance. ΔOQC is a right-angled triangle.
By using pythagoras theorum, $OQ^2 + CQ^2 = r^2$
$4^2 + x^2 = r^2$
or, $16 + x^2 = r^2$ ------(1)

**Step 4**

Similarly, ΔOPB is a right-angled triangle,
$OP^2 + BP^2 = r^2$
$2^2 + (2x)^2 = r^2$
or, $4 + 4x^2 = r^2$ ------(2)

**Step 5**

Subtracting equation (1) from equation (2), we get:
$(4 - 16) + (4x^2 - x^2) = 0$
or, $3x^2 = 12$

or, $x^2 = \frac{12}{3}$

**Step 6**

On substituting $x^2 = \frac{12}{3}$ in equation (1), we get:

$16 + \frac{12}{3} = r^2$

or, $r^2 = \frac{3 \times 16 + 12}{3} = \frac{60}{3}$

**Step 7**

Therefore, area of the circle = $\pi r^2 = 3 \times \frac{60}{3} = 60\ cm^2$

**(13)** 30°

**Step 1**

Angle ∠ADB = ∠ACB [ Angels inscribed by same chord AB]

**Step 2**

Angle ∠ADB = 180° - (∠BAD + ∠ABD ) [ Angels of triangle ABD]

**Step 3**

On equating RHS of above equations
∠ACB = 180° - (∠BAD + ∠ABD )

**Step 4**

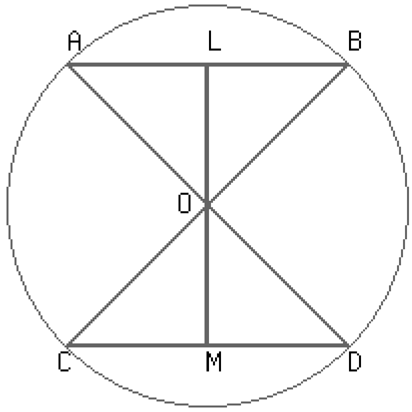
Now replace the values of ∠BAD and ∠ABD in above equations and solve for ∠ACB
∠ACB = 30°

**(14)** True

**Step 1**

AB and CD are two chords that are at a distance of 16 cm from the center.




**Step 2**

Here, OA = OB = OC = OD = r, where r is the radius of the circle. Now consider ΔOCM and ΔOAL.
Here, OC = OA (Radius of the circle)
OL = OM = 16 cm (Given)
As, the distance of a chord from the centre is the perpendicular drawn from the centre on the chord. So,
$\angle OLA = \angle OMC = 90°$
Hence, ΔOAL ≅ ΔOCM by the property of RHS.

**Step 3**

As, ΔOAL ≅ ΔOCM, AL = CM by CPCT.

**Step 4**

Similarly, we can say that ΔOLB ≅ ΔOMD. Hence, LB = MD by CPCT.

**Step 5**

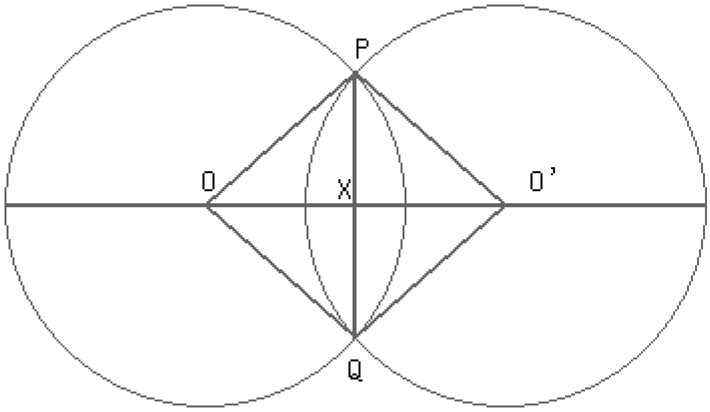
Now, from step 3 and step 4 we have LB = MD and AL = CM. Adding both the equations. We have,
AL + LB = CM + MD
⇒ AB = CD

**Step 6**

This means that the length of the chords are equal. Hence, the given statement is **true**.

**(15)** True

**Step 1**




**Step 2**

We are given that the circles are congruent, which means they have the same radius. We can say that OP = O'P = OQ = O'Q = r, where r is the radius of the circle.

**Step 3**

Now, consider ΔOPQ and ΔO'PQ,
we have OP = O'P (Radius)
OQ = O'Q (Radius)
PQ = PQ (common side)
Hence, ΔOPQ ≅ ΔOPQ by the property of SSS.

**Step 4**

As, ΔOPQ ≅ ΔOPQ, we can say that ∠POQ = ∠PO'Q by CPCT. Hence, the statement is **true**.