

UNIT
3

Rational Numbers

What You'll Learn

How to

- Identify positive and negative decimals and fractions as rational numbers
- Compare and order rational numbers
- Add, subtract, multiply, and divide rational numbers
- Solve problems that involve rational numbers
- Apply the order of operations with rational numbers

Why It's Important

Rational numbers are used by

- building contractors to measure and to estimate costs
- chefs to measure ingredients, plan menus, and estimate costs
- investment professionals to show changes in stock prices

Key Words

fraction

equivalent fraction

numerator

denominator

common denominator

multiple

common multiple

integer

decimal

repeating decimal

terminating decimal

rational number

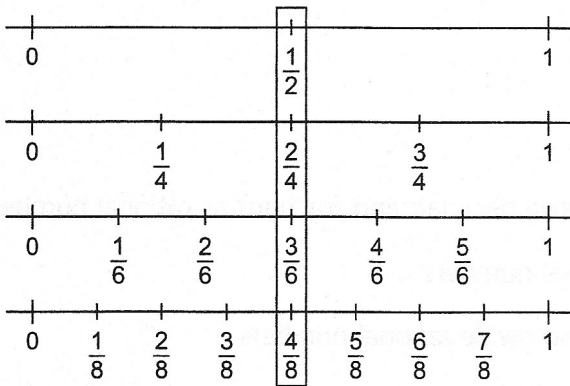
reciprocal

3.1 Skill Builder

Equivalent Fractions

$\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are **equivalent fractions**.

They represent the same distance on a number line.



Here is one way to find equivalent fractions.

Multiply or divide the numerator and denominator by the same number.

$$\frac{1}{2} \xrightarrow{\times 3} \frac{3}{6} \quad \frac{4}{8} \xrightarrow{\div 2} \frac{2}{4}$$

Multiplying or dividing both the numerator and denominator by the same number is like multiplying or dividing by 1. The original quantity is unchanged.

Check

1. Write 2 equivalent fractions.

a) $\frac{7}{10} \xrightarrow{\times 2} \frac{7}{10} = \frac{\quad}{\quad} \xrightarrow{\times 2} \frac{\quad}{\quad}$

b) $\frac{12}{15} \xrightarrow{\quad} \frac{12}{15} = \frac{\quad}{\quad} \xrightarrow{\quad} \frac{\quad}{\quad}$

2. Write an equivalent fraction with the given denominator.

a) $\frac{3}{5} = \frac{\quad}{20}$ $5 \times 4 = 20$, so multiply the numerator and denominator by 4.

b) $\frac{1}{4} = \frac{\quad}{12}$ $4 \times \quad = 12$, so \quad the numerator and denominator by \quad .

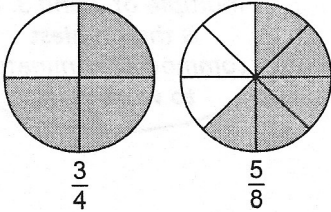
c) $\frac{\quad}{15} = \frac{2}{3}$ $15 \div \quad = 3$, so divide the numerator and denominator by \quad .

d) $\frac{\quad}{24} = \frac{5}{6}$ $24 \div \quad = 6$, so \quad the numerator and denominator by \quad .

Comparing Fractions

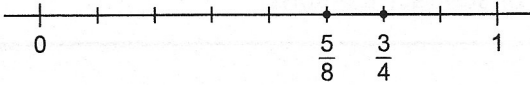
Here are 3 ways to compare $\frac{3}{4}$ and $\frac{5}{8}$.

- Using area models:



Compare the shaded areas: $\frac{3}{4} > \frac{5}{8}$

- Using number lines:



From the number line: $\frac{5}{8} < \frac{3}{4}$

Numbers increase
from left to right on a
number line.

- Writing equivalent fractions:

$$\frac{3}{4} \overset{\times 2}{=} \frac{6}{8}$$

$$\frac{5}{8} < \frac{6}{8}, \text{ so, } \frac{5}{8} < \frac{3}{4}$$

Check

Compare the fractions in each pair. Write $>$, $<$, or $=$.

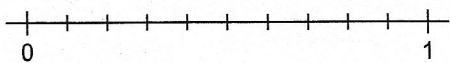
1. a) $\frac{7}{8} - \frac{3}{4}$

b) $\frac{3}{5} - \frac{7}{10}$

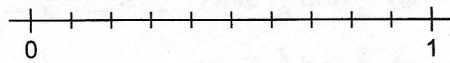
c) $\frac{7}{12} - \frac{2}{3}$

d) $\frac{6}{7} - \frac{6}{8}$

2. a) $\frac{2}{5} - \frac{3}{10}$



b) $\frac{3}{5} - \frac{9}{10}$



Common Denominators

To find a common denominator of $\frac{1}{2}$ and $\frac{2}{3}$:

Look for equivalent fractions with the same denominator.

List the multiples of 2: 2, 4, **6**, 8, 10, 12, 14, ...

List the multiples of 3: 3, **6**, 9, 12, 15, ...

Rewrite $\frac{1}{2}$ and $\frac{2}{3}$ with denominator 6.

$$\begin{array}{ccc} \times 3 & & \times 2 \\ \frac{1}{2} = \frac{3}{6} & & \frac{2}{3} = \frac{4}{6} \\ \times 3 & & \times 2 \end{array}$$

6 is the least common multiple of 2 and 3. It is the simplest common denominator to work with.

Equivalent fractions help us compare, add, or subtract fractions.

Check

1. Write equivalent fraction pairs with a common denominator.

a) $\frac{1}{2}$ and $\frac{3}{8}$

Multiples of 2: 2, 4, 6, 8, 10, ...

Multiples of 8: 8, 16, ...

A common denominator is ____.

So, $\frac{1}{2} = \frac{\quad}{\quad}$ and $\frac{3}{8} = \frac{\quad}{\quad}$

b) $\frac{3}{4}$ and $\frac{5}{6}$

Multiples of 4: _____

Multiples of 6: _____

So, $\frac{3}{4} = \frac{\quad}{\quad}$ and $\frac{5}{6} = \frac{\quad}{\quad}$

c) $\frac{3}{5}$ and $\frac{2}{3}$

Multiples of ____: _____

Multiples of ____: _____

So, $\frac{3}{5} = \frac{\quad}{\quad}$ and $\frac{2}{3} = \frac{\quad}{\quad}$

2. Compare each pair of fractions from question 1.

a) $\frac{1}{2}$ and $\frac{3}{8}$. Since $\underline{\quad} > \underline{\quad}$, $\frac{1}{2} > \frac{3}{8}$

b) $\frac{3}{4}$ and $\frac{5}{6}$. Since $\underline{\quad} < \underline{\quad}$, $\frac{3}{4} < \frac{5}{6}$

c) $\frac{3}{5}$ and $\frac{2}{3}$. Since $\underline{\quad} < \underline{\quad}$, $\frac{3}{5} < \frac{2}{3}$

Converting between Fractions and Decimals

- Fractions to decimals

The fraction bar represents division. For example:

$$\frac{1}{6} \text{ means } 1 \div 6$$

Use a calculator:

$$1 \div 6 = 0.166\ 666\dots$$

$$= 0.1\bar{6}$$

$$\text{So, } \frac{1}{6} = 0.1\bar{6}$$

$0.1\bar{6}$ is a **repeating decimal**.

The bar over the 6 means that 6 repeats.

$$\frac{7}{8} \text{ means } 7 \div 8$$

Use a calculator:

$$7 \div 8 = 0.875$$

$$\text{So, } \frac{7}{8} = 0.875$$

0.875 is a **terminating decimal**.

- Decimals to fractions

Use place value. For example:

0.7 means 7 tenths.

$$\text{So, } 0.7 = \frac{7}{10}$$

0.23 means 23 hundredths

$$\text{So, } 0.23 = \frac{23}{100}$$

Check

1. Write each fraction as a decimal.

a) $\frac{3}{4} = 3 \div 4$

= _____

b) $\frac{2}{3} =$ _____

= _____

c) $\frac{5}{8} =$ _____

= _____

d) $\frac{5}{9} = 5 \div 9$

= _____

e) $4\frac{1}{5} = 4 + \frac{1}{5}$

= 4 + _____

= 4 + _____

= _____

f) $2\frac{1}{3} = 2 +$ _____

= 2 + _____

= 2 + _____

= _____

2. Which numbers in question 1 are:

a) repeating decimals? _____

b) terminating decimals? _____

3. Write each decimal as a fraction.

a) 0.3 = _____

b) 0.9 = _____

c) 0.11 = _____

d) 0.87 = _____

e) 1.5 = _____

f) 5.7 = _____

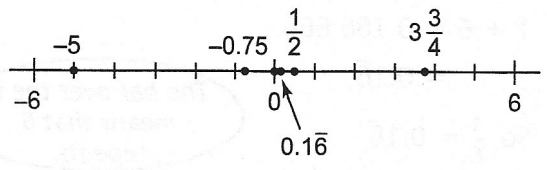
3.1 What Is a Rational Number?

FOCUS Compare and order rational numbers.

Rational numbers include:

- integers
- positive and negative fractions
- positive and negative mixed numbers
- repeating and terminating decimals

Here is a number line that displays some rational numbers.



Example 1 Finding a Rational Number between Two Given Numbers

Find 2 rational numbers between $2\frac{1}{3}$ and $3\frac{3}{4}$.

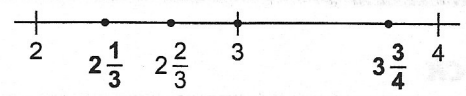
Solution

Label a number line from 2 to 4.

$2\frac{1}{3}$ is one-third of the way from 2 to 3.

$3\frac{3}{4}$ is three-quarters of the way from 3 to 4.

From the number line, 2 rational numbers between $2\frac{1}{3}$ and $3\frac{3}{4}$ are: $2\frac{2}{3}$ and 3



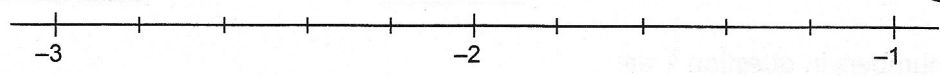
There are many correct solutions. Which ones can you name?

Check

1. Find 2 rational numbers between each pair of numbers.

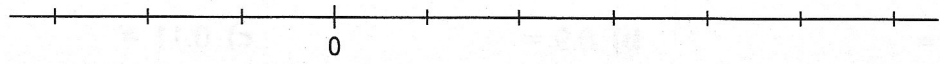
a) $-2\frac{1}{3}$ and $-1\frac{2}{5}$

Plot points to show $-1\frac{2}{5}$ and $-2\frac{1}{3}$.



From the number line, 2 values between $-2\frac{1}{3}$ and $-1\frac{2}{5}$ are: _____ and _____

b) -0.3 and 0.6



From the number line, 2 values between -0.3 and 0.6 are: _____ and _____

Example 2 Comparing Rational Numbers on a Number Line

Order each set of rational numbers from least to greatest.

a) $0.3, 0.\overline{3}, -1.7, 0.6, -0.6$

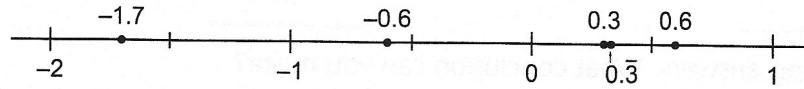
b) $3\frac{1}{4}, -\frac{3}{4}, -\frac{4}{8}, 1\frac{3}{4}, -2\frac{3}{8}$

Solution

a) Plot the numbers on a number line.

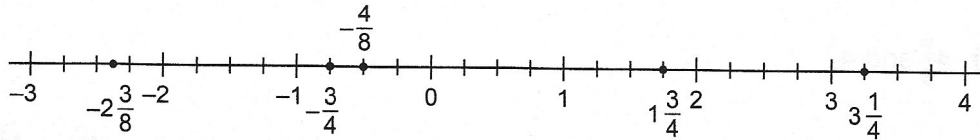
To plot 0.3 and $0.\overline{3}$, think: $0.\overline{3} = 0.3333\dots$

So, $0.\overline{3}$ is slightly greater than 0.3 .



From the number line, the order from least to greatest is: $-1.7, -0.6, 0.3, 0.\overline{3}, 0.6$

b) Plot the numbers on a number line.

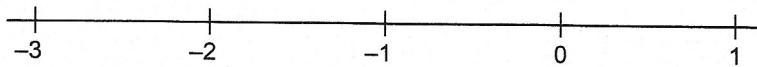


From the number line, the order from least to greatest is: $-2\frac{3}{8}, -\frac{3}{4}, -\frac{4}{8}, 1\frac{3}{4}, 3\frac{1}{4}$

Check

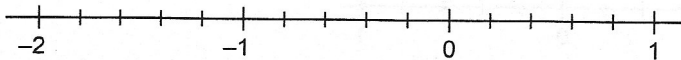
1. Order each set of numbers from least to greatest.

a) $-1.\overline{8}, 0.7, -2, -2.1, -0.3$



From the number line, the order from least to greatest is: _____

b) $-1\frac{9}{10}, -2, -1\frac{4}{5}, \frac{4}{5}, -1\frac{1}{5}$



The number line is divided in fifths to help you plot the numbers.

From the number line, the order from least to greatest is: _____

Practice

1. Write each rational number as a decimal.

a) $\frac{3}{5} = \underline{\quad} \div \underline{\quad}$
 $= \underline{\quad}$

b) $\frac{5}{3} = \underline{\quad}$
 $= \underline{\quad}$

c) $-\frac{3}{5} = -(\underline{\quad} \div \underline{\quad})$
 $= \underline{\quad}$

d) $-\frac{3}{5} = (\underline{\quad}) \div \underline{\quad}$
 $= \underline{\quad}$

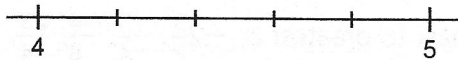
e) $-\frac{5}{3} = (\underline{\quad}) \div \underline{\quad}$
 $= \underline{\quad}$

f) $\frac{3}{-5} = \underline{\quad}$
 $= \underline{\quad}$

Look for matching answers. What conclusion can you make?

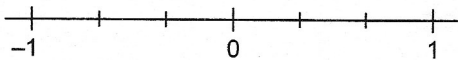
2. Plot and compare each pair of rational numbers.

a) $4\frac{2}{5}$ and $4\frac{3}{5}$



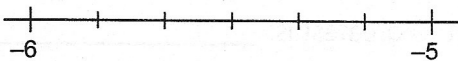
From the number line, $4\frac{2}{5} < 4\frac{3}{5}$

b) $\frac{2}{3}$ and $-\frac{1}{3}$

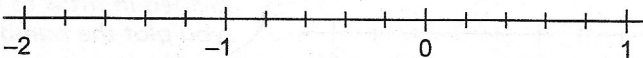


From the number line, $\frac{2}{3} > -\frac{1}{3}$

c) $-5\frac{5}{6}$ and $-5\frac{1}{6}$



3. a) Write a decimal to match each point on the number line.



b) Write the numbers in part a from least to greatest.
