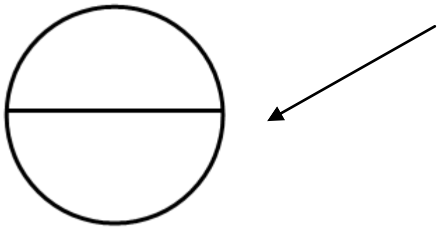
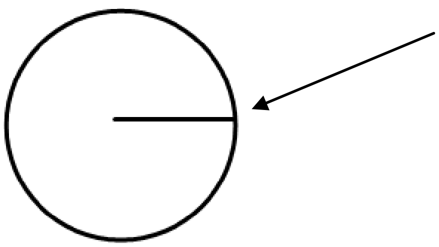
Unit 8: Circle Geometry **Grade 9 Math**

Introduction: Definitions



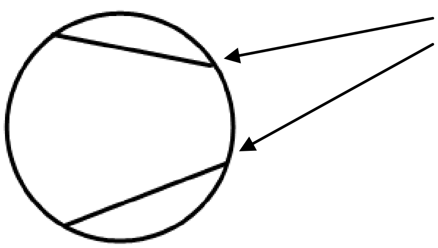
Diameter

the distance across a circle, measured through its center; or the line segment that joins two points on the circle and passes through the center.



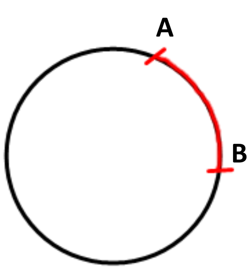
Radius

the distance or line segment from the center of a circle to any point on the circle.



Chord(s)

a line segment that joins two points on a circle.

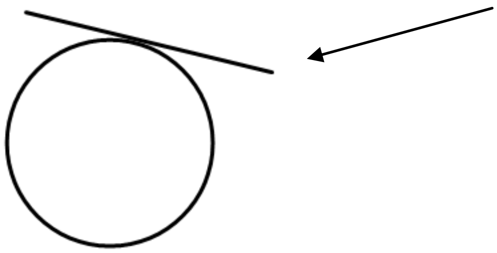


Arc

A segment of the circumference of a circle.

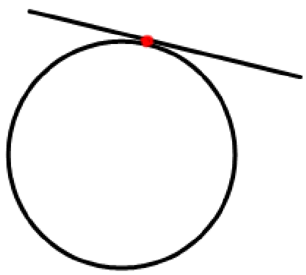
Minor Arc

The shorter of two arcs between two points on a circle.
For example: $\overset{\frown}{AB}$



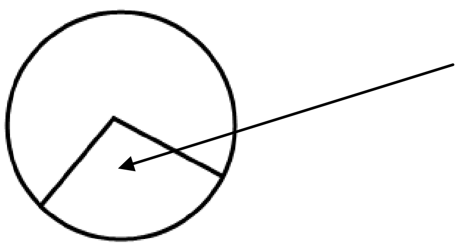
Tangent

a line that intersects a circle at only one point.



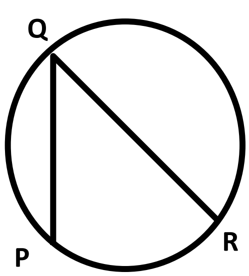
Point of Tangency

the point where a tangent intersects a circle



Central Angle

An angle whose arms are radii of a circle.

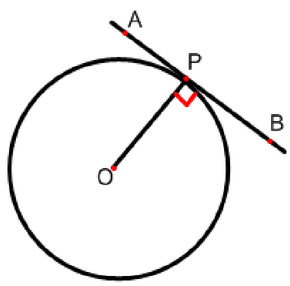


Inscribed Angle

An angle in a circle with its vertex and endpoints of its arms on the circle.

For example, $\angle$ PQR

Section 8.1 Properties of Tangents to a Circle

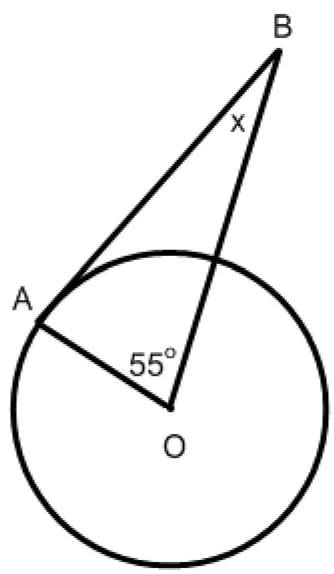


Tangent-Radius Property

A tangent to a circle is perpendicular to the radius at the point of tangency.
$\angle$ APO = $\angle$ BPO = 90^0

Example Problems

A) Point O is the center of a circle and AB is tangent to the circle. In OAB, $\angle$ AOB = 55^0. Determine the measure of $\angle$ OBA.



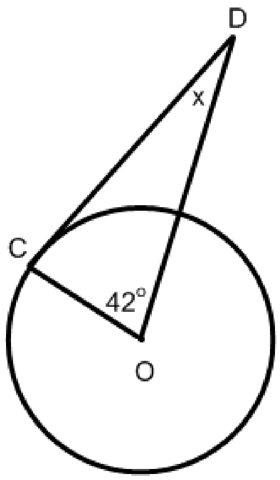
Since $\angle$ A = 90^0 and $\angle$ 0 = 55^0

Then 90 + 55 = 145

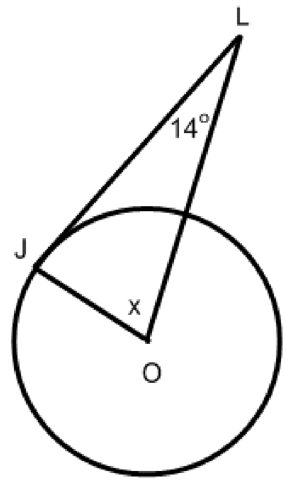
The three angles in a triangle add to 180^0. So $\angle$ x = 180 – 145 = 35^0.

Try to find the missing angles in the following diagrams

A)



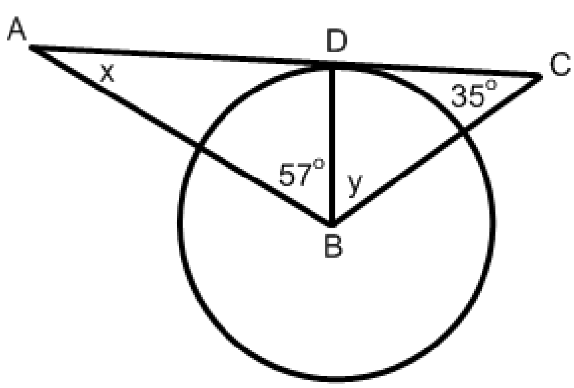
B)



x = 48°

x = 76°

Application Example



Since AC is a tangent …
$\angle$ BDA = $\angle$BDC = 90^0

Find x

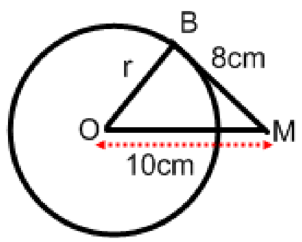
x + 90 + 57 = 180
x + 147 = 180
x + 147 - 147 = 180 - 147
x = 33°

Find y

y + 90 + 35 = 180
y + 125 = 180
y + 125 - 125 = 180 - 125
y = 55°

Using the Pythagorean Theorem in a Circle

1.

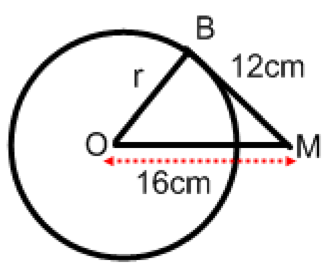


Since BM is a tangent we know that $\angle$ OBM = 90^0.

$a^2 + b^2 = c^2$
$8^2 + b^2 = 10^2$
$64 + b^2 = 100$
$b^2 = 100 – 64$
$b^2 = 36$
$b= \sqrt{36}$
b = 6 cm

Try this one!

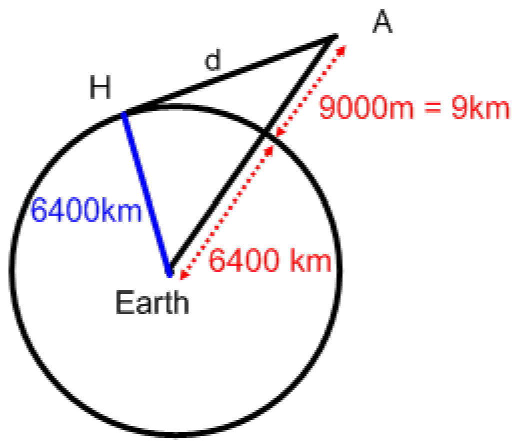
2.



Since BM is a tangent we know that $\angle$ OBM = 90^0.

$a^2 + b^2 = c^2$
$12^2 + b^2 = 16^2$
$144 + b^2 = 256$
$b^2 = 256 – 144$
$b^2 = 112$
$b= \sqrt{112}$
b = 10.6 cm

3. An airplane is cruising at an altitude of 9000m. A cross section of the earth is a circle with a radius approximately 6400km. A passenger wonders how far she is from a point H on the horizon she sees outside the window. Calculate the distance to the nearest kilometer.

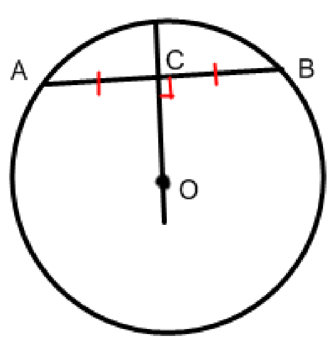


$a^2 + b^2 = c^2$
$d^2 + 6400^2 = 6409^2$
$d^2 + 40960000 = 41075281$
$d^2 = 41075281 - 40960000$
$d^2 = 115281$
$d = \sqrt{115281}$
$d = 339.5$ km

8.2 Properties of Chords in a Circle

In any circle with center O and chord AB:

- If OC bisects AB, then OC $\perp$ AB
- If OC $\perp$ AB, then AC = CB
- The perpendicular bisector of AB goes through the center O.



Remember:

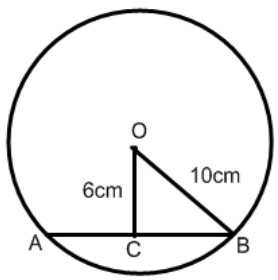
Perpendicular means there is a 90° angle.

Bisector means it is divided into 2 equal parts

If AC = 10cm, then BC =10cm

Example # 1

O is the center of the circle. Find the length of chord AB.



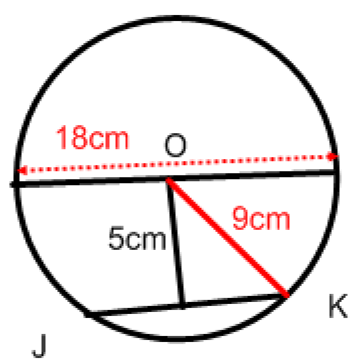
Solution: Use the Pythagorean Theorem to solve for BC

$a^2 + b^2 = c^2$
$6^2 + BC^2 = 10^2$
$36 + BC^2 = 100$
$BC^2 = 100 – 36$
$b^2 = 64$
$BC= \sqrt{64}$
$BC = 8$ cm

AC= BC = 8cm
So the length of AB is 2 x 8cm = 16cm

Example # 2

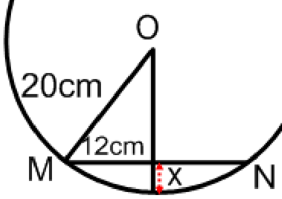
The diameter of a circle is 18cm. A chord JK is 5cm from the center. Find the length of the chord.



$a^2 + b^2 = c^2$
$5^2 + b^2 = 9^2$
$25 + b^2 = 81$
$b^2 = 81 – 25$
$b^2 = 56$
$b= \sqrt{56}$
$b = 7.5$ cm

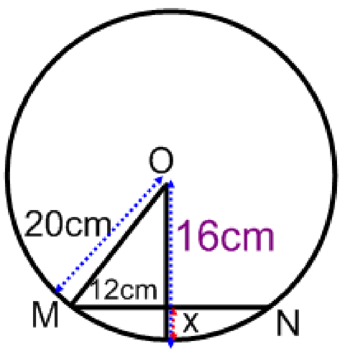
If b = 7.5 cm, then the chord JK is 2 x 7.5cm = 15cm

Example # 3 A chord MN is 24cm. The radius of a circle is 20cm. Find the length of x.



Since the chord is 24cm, half it is 12cm. Use the Pythagorean Theorem to find the missing side of the triangle.

$a^2 + b^2 = c^2$
$12^2 + b^2 = 20^2$
$144 + b^2 = 400$
$b^2 = 400 – 144$
$b^2 = 256$
$b = \sqrt{256}$
$b = 16$ cm



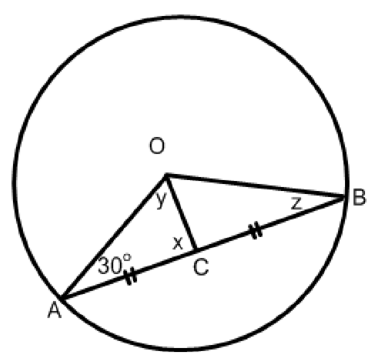
Radius is 20 cm ALL the way around the circle!

The length of x must be 20 cm – 16 cm = 4 cm

Example # 4:

Finding Angle Measurements

x , y and z.



Solution

Since OC bisects chord AB, OC is perpendicular to AB. Therefore, $x = 90^o$

The 3 angles in a triangle must add up to 180^o.

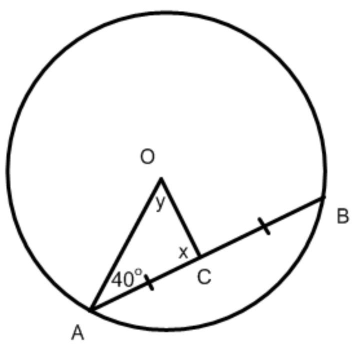
$y + 30 + 90 = 180$
$y + 120 = 180$
$y + 120 - 120 = 180 - 120$
$y = 60^o$

Since radii are equal (OA = OB) and ΔOAB is isosceles, $z = 30^o$.

Remember that in an isosceles triangle the 2 base angles are equal.

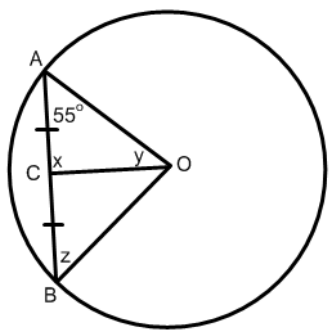
Try These

A).



$x = 90^o$ and $y = 50^o$

B).



$x = 90^o$, $y = 35^o$ and $z = 55^o$

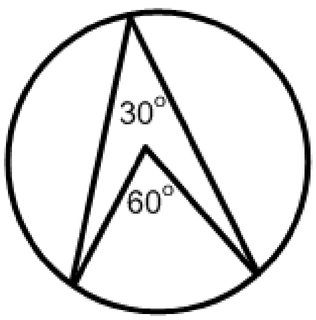
Section 8.3 Properties of Angles in a Circle

Central Angle and Inscribed Angle Property

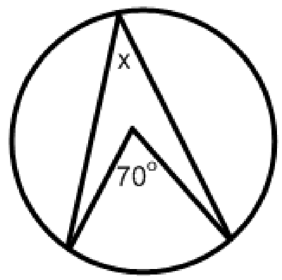
The measure of a central angle is twice the measure of an inscribed angle subtended by the same arc.

Examples

#1

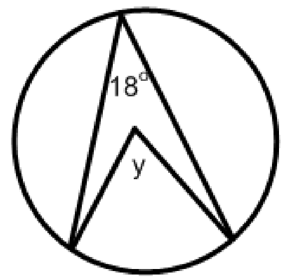


#2



$x = 35^0$

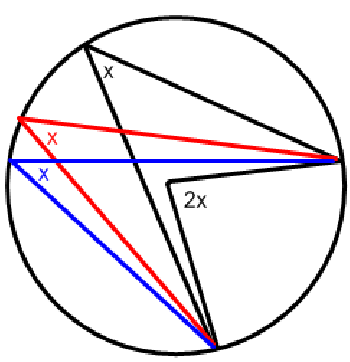
#3



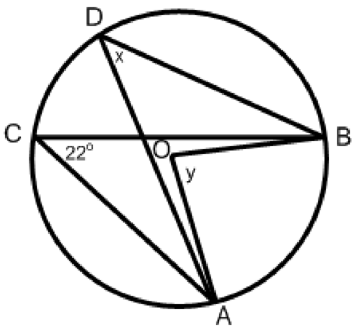
$y = 36^0$

Inscribed Angles Property

Inscribed angles subtended by the same arc are equal.



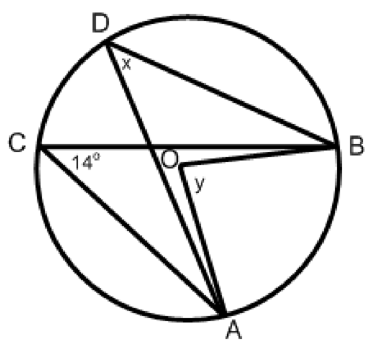
Examples #1



$\angle$ ACB and $\angle$ ADB are inscribed angles subtended by the same arc AB. So,
$\angle$ ACB = $\angle$ ADB. $x = 22^0$.

Central angle $\angle$ AOB and inscribed angle $\angle$ ACB are both subtended by arc AB. $\angle$ AOB = 2 × $\angle$ ACB
$y = 2 \times 22$ $y = 44^0$

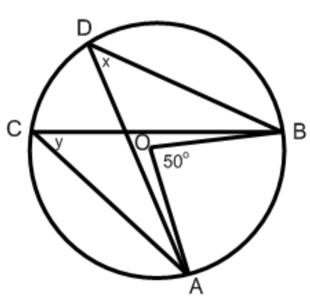
#2



$\angle$ ACB and $\angle$ ADB are inscribed angles subtended by the same arc AB. So,
$\angle$ ACB = $\angle$ ADB. $x = 14^0$.

Central angle $\angle$ AOB and inscribed angle $\angle$ ACB are both subtended by arc AB. $\angle$ AOB = 2 × $\angle$ ACB
$y = 2 \times 14$ $y = 28^0$

#3



Since both inscribed angles are subtended from the same arc as the central angle

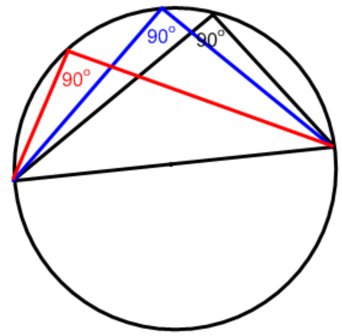
$\angle ACB = \angle ADB = \frac{1}{2} \angle AOB.$

$y = x = \frac{1}{2} (50^0)$

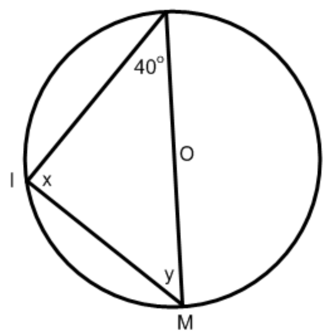
$y = x = 25^0$

Angles in a Semicircle Property

Inscribed angles subtended by a semicircle (half the circle) are right angles. This means these angles use the diameter.



Example #1 Find the missing angle measures.

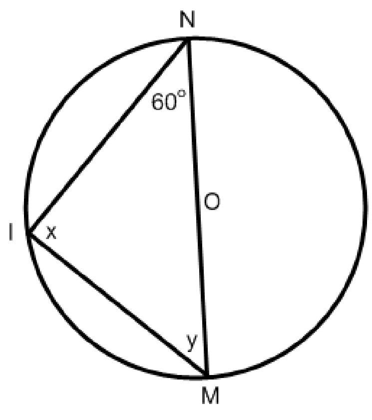


$\angle$MIN is an inscribed angle subtended by a semicircle.
So, $x = 90^0$.

Since three angles in a triangle add to 180^0,

$y + 90 + 40 = 180$
$y + 130 = 180$
$y + 130 – 130 = 180 – 130$
$y = 50^0$

Example # 2 - Try this one!



$\angle$MIN is an inscribed angle subtended by a semicircle.
So, $x = 90^0$.

Since three angles in a triangle add to 180^0,

$y + 90 + 60 = 180$
$y + 150 = 180$
$y + 150 – 150 = 180 – 150$
$y = 30^0$