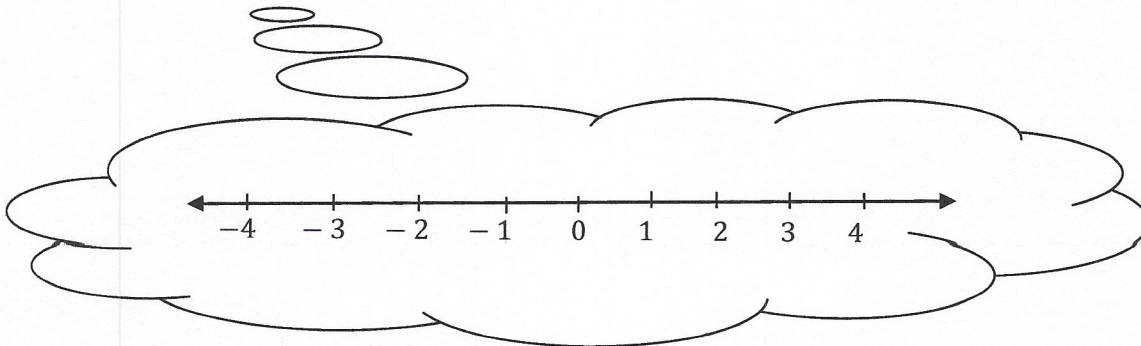## Section 3.1: What is a Rational Number?

Integers, I, is a set of numbers that include positive and negative numbers and zero.

Imagine a number line

These numbers are all integers. The set of integers does not include decimals or fractions.

Rational Numbers, Q, is any number that can be written in the form, $\dfrac{m}{n}$,

where m and n are both integers but $n \neq 0$.

Example: Using any two integers create a fraction and change to a decimal.

1a). $\dfrac{-6}{3} = \dfrac{-2}{1} = -2$ ***   notice -2 is an integer and a rational number.

***   any integer can be written as a fraction using 1 as the denominator.

b). $\dfrac{2}{3} = 0.\overline{6}$ *** $0.\overline{6}$ is a repeating decimal and a rational number.

c). $\dfrac{7}{8} = 0.875$ *** 0.875 is a terminating decimal and a rational number.

d). $\dfrac{100}{25} = 4$ *** 4 is an integer and a rational number.

Therefore, rational numbers include all integers, fractions, terminating decimals and repeating decimals.

2. Identify the rational numbers below:

a). $\dfrac{-1}{4}$ ⟶ Rational. It's a fraction. Even as a terminating decimal, -0.25 it's still rational.

b). $\sqrt{9}$ ⟶ is 3. Rational. 3 is an integer.

c). $\dfrac{-4}{-9}$ ⟶ $\dfrac{4}{9}$ is rational, it's a fraction. Even as a repeating decimal, $0.\overline{4}$ it's still rational.

d). $\sqrt{75}$ ⟶ = 8.660254038... ⎫
                                      ⎬ These numbers are non-repeating and non-terminating decimals.
e). π ⟶ = 3.1415926535... ⎭

These types of numbers are called **irrational numbers**, $\overline{Q}$.

p.101 # 6, 7 and 12

Compare and Order Rational Numbers

1. Use > , < , or = to determine which rational number is greater, where possible.

a). $\dfrac{4}{7} \rangle \dfrac{5}{9}$

* use a common denominator
    ...63

$\dfrac{4}{7} \dfrac{\times 9}{\times 9}$ $\dfrac{5}{9} \dfrac{\times 7}{\times 7}$

$\dfrac{36}{63} \rangle \dfrac{35}{63}$

* Larger numerator represents the greater fraction.

b). $\dfrac{-3}{8} \rangle \dfrac{-5}{8}$

* already has a common denominator so look at the numerators.

With negative numbers closer to zero is greater,

since $-3 > -5$ then $\dfrac{-3}{8} \rangle \dfrac{-5}{8}$

c). $\dfrac{2}{7} > \dfrac{2}{9}$

*   for two positive fractions which have common numerators, the smallest denominator is the greater fraction.

d). $\dfrac{-2}{7} < \dfrac{-2}{9}$

*   for negative fractions which have common numerators, the larger denominator is the greater fraction.

e). $\dfrac{-3}{4} < \dfrac{3}{4}$

*   positive is always greater than negative.

*   these fractions are called **opposites.**

    For every positive fraction, or decimal, there is a corresponding negative fraction or decimal.

    ($\dfrac{2}{5}$ and $\dfrac{-2}{5}$ are opposites)

    (0.25 and −0.25 are opposites).

f). $\dfrac{-10}{4} > -2.8$

*   change $\dfrac{-10}{4}$ to a decimal or change −2.8 to a fraction.

$$ -10 \;\longrightarrow\; \begin{array}{r} 2.5 \\ 4\,\overline{)10.0} \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array} $$
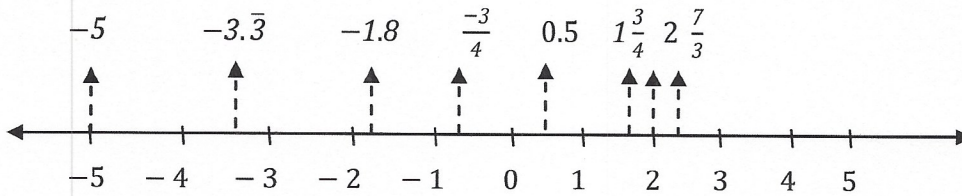
$$-2.5$$

And $-2.5 > -2.8$

g). $\dfrac{-7}{8} = \dfrac{7}{-8}$ *   Regardless of the position of the negative sign, these fractions are equal. A positive divided by a negative is always negative.

$$\dfrac{-7}{8} = \dfrac{7}{-8} = -\dfrac{7}{8}$$

2. Place these rational numbers in descending order.

$$\frac{-3}{4} \ , \ 0.5 \ , -1.8 \ , -5 \ , \ \frac{7}{3} \ , \ 2 \ , -3.\overline{3} \ , \ 1\frac{3}{4}$$

Descending Order (from largest to smallest)

$$\frac{7}{3} \ , \ 2 \ , \ 1\frac{3}{4} \ , \ 0.5 \ , \ \frac{-3}{4} \ , -1.8 \ , -3.\overline{3} \ , -5$$

Writing a Rational Number between two given numbers.

1. Identify a decimal between each pair of rational numbers.

a). $\frac{-1}{2}$ and $\frac{-1}{4}$ $\longrightarrow$ -0.3

b). -0.25 and -0.26 $\longrightarrow$ -0.255

2. Identify a fraction between each pair of rational numbers.

a). $\frac{-2}{3}$ and $\frac{-3}{4}$ $\longrightarrow$ $\frac{-7}{10}$

$\frac{-2}{3} = -0.\overline{6}$ and $\frac{-3}{4} = -0.75$

Need a fraction that falls between $-0.\overline{6}$ and -0.75, try

$$-0.7 = \frac{-7}{10}$$

b). $\frac{5}{2}$ and $\frac{7}{3}$ $\longrightarrow$ $\frac{24}{10} = \frac{12}{5}$

$\frac{5}{2} = 2.5$ and $\frac{7}{3} = 2.\overline{3}$

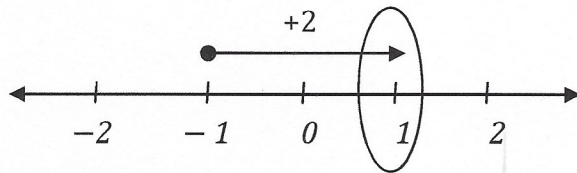Need a fraction that falls between 2.5 and $2.\overline{3}$, try

$2.4 = \frac{24}{10}$

Section 3.2: Adding Rational Numbers
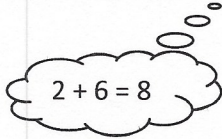
Adding Integers:

a). $(-1) + (+2)$ ——————→   let's try using a number line (from gr. 7)

- start at the first integer
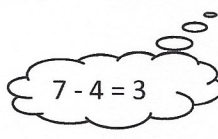- go left for adding a positive
- go right for adding a negative

Answer = +1

b). $(-2) + (-6) = -8$

2 + 6 = 8

c). $(-7) + (+4) = -3$

7 - 4 = 3

Try These!

1). $(-2) + (-1)$

2). $-6 + (-4)$

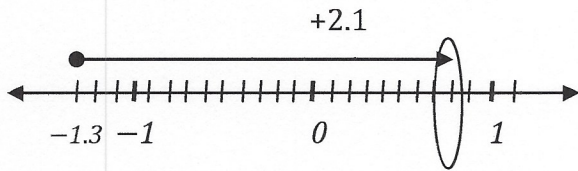3). $(+8) + (-12)$

4). $(+5) + (-19)$

5). $(-5) + 3 + (-9)$

6). $7 + (-2) + (-7) + (+4)$

Answers: a). -3 b). -10 c). -4 d). -14 e). -11 f). +2

Adding Decimals:

1. $(-1.3) + (+2.1)$ ──────────▶ let's try using a number line

+2.1

Answer = +0.8

2. Write an addition equation for:

1.2

1.9
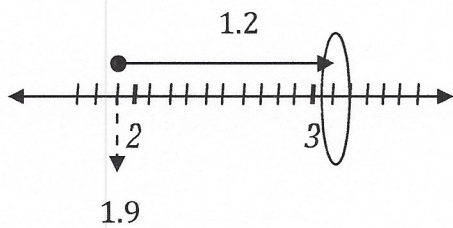
Answer : $(+1.9) + (+1.2) = 3.1$

Add.

a). $(-2.8) + (-6.5) = -9.3$

$2.8 + 6.5 = 9.3$

c). $(-7.3) + (+3.1) = -4.2$

$7.3 - 3.1 = 4.2$

Try These!

1). $(+2.4) + (-1.7)$

2). $(-3.5) + 6.3$

3). $(-4.1) + (-3.1)$

4). $0.67 + (-0.83)$

5). $-1.5 + 1.25$

6). $-0.583 + 0.625$

Answers: a). 0.7 b). 2.8 c). -7.2 d). -0.16 e). -0.25 f). 0.042

Adding Fractions:

a). $\dfrac{-7}{9} + \dfrac{5}{9} = \dfrac{-2}{9}$ b). $\dfrac{2}{5} + \dfrac{-3}{5} = \dfrac{-1}{5}$

***   These fractions already have common denominators (the same bottom #) so just add the numerators (top #'s)

c). $\dfrac{-7}{8} + \dfrac{3}{4}$ * get common denominators first (make the bottom # the same)

$\dfrac{-7}{8} + \dfrac{3^{\times 2}}{4_{\times 2}}$ * multiply the numerator and denominator by the same #

$\dfrac{-7}{8} + \dfrac{6}{8} = \dfrac{-1}{8}$

b). $-3\dfrac{1}{3} + 2\dfrac{5}{6}$ * change to an improper fraction first

$\dfrac{-10}{3} + \dfrac{17}{6}$ * get common denominators

$\dfrac{-10^{\times 2}}{3_{\times 2}} + \dfrac{17}{6}$

$\dfrac{-20}{6} + \dfrac{17}{6} = \dfrac{-3}{6} = \dfrac{-1}{2}$ * in lowest terms.

****   Always reduce your fraction answer to lowest terms.

Try These!

a). $1\dfrac{1}{2} + \left(-2\dfrac{1}{3}\right)$ b). $\dfrac{3}{8} + \dfrac{7}{6}$ c). $\dfrac{-3}{2} + \dfrac{1}{6}$

Answers: a). $\dfrac{-5}{6}$ b). $\dfrac{37}{24} = 1\dfrac{7}{24}$ c). $\dfrac{-8}{6} = \dfrac{-4}{3}$

Addition Word Problems

1. A guardrail needs to be exactly 19.77 m long. A contractor has 3 pieces measuring 2.21m, 9.14m an 3.21m, does he have enough to complete the guardrail?

   Answer: 2.21 + 9.14 + 3.21 = 14.56   No, he does not have enough.

2. Peter estimates that it takes him $\frac{1}{4}$ h to prepare the dough, $\frac{1}{10}$ h to grate the cheese, $\frac{1}{3}$ h to prepare the toppings, and $\frac{2}{5}$ h to bake the pizza.

   a). What fraction of time did it take Peter in total to prepare the pizza?

   Answer: $\frac{1}{4} + \frac{1}{10} + \frac{1}{3} + \frac{2}{5}$

   $$\frac{1^{\times15}}{4^{\times15}} + \frac{1^{\times6}}{10^{\times6}} + \frac{1^{\times20}}{3^{\times20}} + \frac{2^{\times12}}{5^{\times12}}$$

   $$\frac{15}{60} + \frac{6}{60} + \frac{20}{60} + \frac{24}{60} = \frac{65}{60} = 1\frac{5}{60} = 1\frac{1}{12}$$

   b). What was the actual time it took to prepare the pizza?

   $1\frac{5}{60}$ means 1 hour and 5 minutes.

Addition Practice Questions

1. $(+3.5) + (-4.2)$ 2. $(-2.2) + (-1.6)$ 3. $(-0.17) + 0.83$

4. $\frac{3}{6} + \frac{1}{6}$ 5. $-\frac{5}{6} + \frac{5}{9}$ 6. $-2\frac{1}{4} + \frac{5}{8}$

Answers: 1. -0.7 2. -3.8 3. 0.66 4. $\frac{2}{3}$ 5. $\frac{-5}{18}$ 6. $\frac{-13}{8}$

Section 3.3 Subtracting Rational Numbers

To subtract rational numbers we **ADD THE OPPOSITE**. Every subtraction problem can be rewritten as an addition problem.

Integer Examples:

a). $(+5) - (+3)$
$(+5) + (-3)$
$+2$

b). $7 - (-4)$
$7 + (+4)$
$+11$

c). $-4 - (-2) - (+3)$
$-4 + (+2) + (-3)$
$-2 + (-3) = -5$

Decimal Examples:

a). $(+0.23) - (-1.46)$
$(+0.23) + (+1.46)$
1.69

b). $(-1.39) - (+2.41)$
$(-1.39) + (-2.41)$
-3.80

Fraction Examples:
   *** we still need common denominators to subtract fractions.
   *** we still need to change mixed numbers to improper fractions.

a). $\dfrac{5}{7} - \dfrac{-3}{7}$

$\dfrac{5}{7} + \dfrac{+3}{7} = \dfrac{8}{7}$

b). $-1\dfrac{1}{4} - (-2\dfrac{2}{3})$

$-\dfrac{5}{4} + (+\dfrac{8}{3}) = -\dfrac{5 \times 3}{4 \times 3} + (+\dfrac{8 \times 4}{3 \times 4})$

$-\dfrac{15}{12} + (+\dfrac{32}{12}) = \dfrac{17}{12} = 1\dfrac{5}{12}$

Whenever there is a negative fraction, use the negative sign with the numerator.

Try These!

1. $(-8.93) - (+1.25)$

2. $3.34 - (-1.16)$

3. $\dfrac{-4}{5} - (\dfrac{-1}{2})$

4. $1\dfrac{1}{6} - \dfrac{3}{4}$

Answers: 1. 10.18   2. 4.5   3. $\dfrac{-3}{10}$ 4. $\dfrac{5}{12}$

Subtraction Word Problems

1. The temperature in St. John's in 6.5^0C. In Corner Brook it is 8^0C colder. What is the temperature in Corner Brook?

   Answer: $6.5 - 8 = 6.5 + (-8) = -1.5^0$C

2. A piece of pipe is 146.3 cm long. A piece 13.7 cm is cut off. How long is the remaining piece?

   Answer: $146.3 - 13.7 = 146.3 + (-13.7) = 132.6$ cm

3. A person climbs $12\frac{2}{3}$ meters above the water to the top of a cliff. He dives into the water and reaches $-3\frac{1}{6}$ meters below the surface. What is the difference in these heights?

   Answer: $12\frac{2}{3} - (-3\frac{1}{6}) = \frac{38}{3} + (+\frac{19}{6})$

   $= \frac{38 \times 2}{3 \times 2} + \frac{19}{6} = \frac{76}{6} + \frac{19}{6} = \frac{95}{6} = 15\frac{5}{6}$ meters

4. Which expression has the same answer as $-2.3 - (-3.9)$?

   a). $-2.3 + (-3.9)$ b). $2.3 - (-3.9)$ c). $-2.3 - (+3.9)$ d). $-2.3 + (+3.9)$

   Answer: D

5. Determine the missing number in each subtraction equation.

   a). $2.5 - \boxed{} = 3.8$ b). $\boxed{} - \frac{-3}{10} = \frac{2}{5}$

   Answers: a). -1.3 b). $\frac{1}{10}$

Section 3.4: Multiplying Rational Numbers

When multiplying or dividing rational numbers, the rules for the positive and negative signs are the same as with integers.

<table>
<tr><td colspan="2" align="center">Multiplying and Dividing</td></tr>
<tr><td>+ and + = +
− and − = +</td><td>same signs is
POSITIVE</td></tr>
<tr><td>− and + = −
is
+ and − = −</td><td>opposite signs

NEGATIVE</td></tr>
</table>

Multiplying Integers

*** Be careful of the signs.

a). $(-6) \times (-3) = 18$

b). $20 \times (-2) = -40$

Multiplying Decimals

To multiply decimals without a calculator, line-up the last decimal place. The number with the most digits should go on top. Don't worry about the sign until your final answer.

a). $(-1.5) \times 1.8 = ?$ *This is negative*

Workings:
$$
\begin{array}{r}
1.5 \\
\times\ 1.8 \\
\hline
120 \\
+\ 150 \\
\hline
2.70
\end{array}
$$

Move the decimal in two places in the final answer.

$$
\begin{array}{r}
1.5 \\
\times\ 1.8
\end{array}
$$

Answer: − 2.70

b). $(- 2.6) \times (- 3.25) =$ *This is positive*

Workings:
$$
\begin{array}{r}
3.25 \\
\times\ 2.6 \\
\hline
1950 \\
+\ 6500 \\
\hline
8.450
\end{array}
$$

Move the decimal in three places in the final answer.

$$
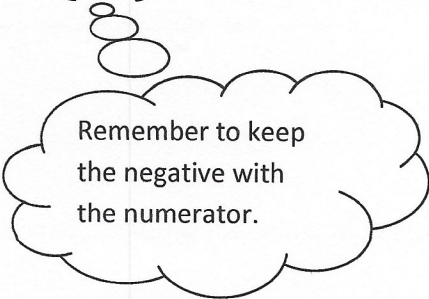\begin{array}{r}
3.25 \\
\times\ 2.6
\end{array}
$$

Answer : + 8.450

Multiplying Fractions

To multiply fractions, multiply straight across. $\dfrac{\text{Numerator} \times \text{Numerator}}{\text{Denominator} \times \text{Denominator}}$

a). $\left(-\dfrac{2}{5}\right) \times \dfrac{3}{8} \;=\; \dfrac{-2 \times 3}{5 \times 8} \;=\; \dfrac{-6}{40}$

Remember to keep the negative with the numerator.

Reduce Lowest terms $\dfrac{-6 \div 2}{40 \div 2} = \dfrac{-3}{20}$

b). $2\dfrac{1}{4} \times \left(-\dfrac{2}{3}\right)$ ** change mixed numbers to improper.

$\dfrac{9}{4} \times \dfrac{-2}{3} \;=\; \dfrac{-18}{12} \;=\; \dfrac{-3}{2}$ in lowest terms

c). $3 \times \dfrac{5}{8}$ change $3 = \dfrac{3}{1}$ $\dfrac{3}{1} \times \dfrac{5}{8} = \dfrac{15}{8}$

** when multiplying a fraction by a whole number, write the whole number as a fraction over one.

d). $\dfrac{3}{5} \times \;?\; = \dfrac{-6}{25}$

Missing Fraction: $\dfrac{-2}{5}$
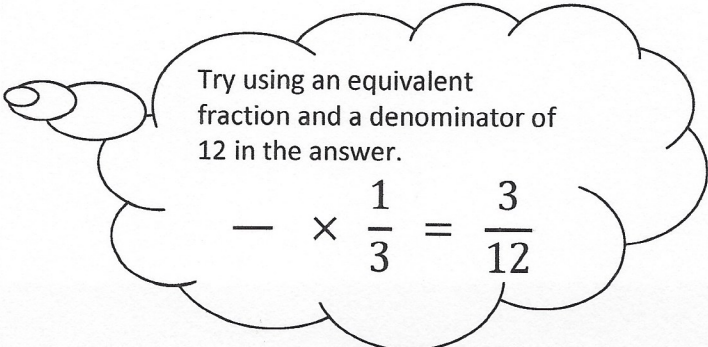
Challenge

e). $\;?\; \times \dfrac{1}{3} = \dfrac{1}{4}$

Missing fraction: $\dfrac{3}{4}$

Try using an equivalent fraction and a denominator of 12 in the answer.

$-\!\!- \times \dfrac{1}{3} = \dfrac{3}{12}$

Always reduce answers to lowest terms.

The most common way is to multiply first then simplify the answer.

There is another way!

You can simplify the fractions first before you multiply.

Examples.

Determine each product. Be sure to simplify your answer.

$$\frac{-11}{7} \times \frac{-21}{44}$$

** it would be easier to reduce first before multiplying since the numbers are so big.

** because we are multiplying, you can reduce either numerator with either denominator.

Can $\frac{-11}{7}$ reduce?   No, so try the other denominator.

Now try $\frac{-11}{44}$. Can this reduce? Yes. $\frac{\cancel{-11}^{-1}}{7} \times \frac{-21}{\cancel{44}_4} \implies \frac{-1}{7} \times \frac{-21}{4}$

Can $\frac{-21}{4}$ reduce ? No, so try the other denominator.

Now try $\frac{-21}{7}$. Can this reduce? Yes. $\frac{-1}{\cancel{7}_1} \times \frac{\cancel{-21}^{-3}}{4} \implies \frac{-1}{1} \times \frac{-3}{4} = \frac{3}{4}$

in lowest terms

If you are uncomfortable with this way you can always multiply first and reduce the final answer.

$$\frac{-11}{7} \times \frac{-21}{44} = \frac{231}{308} \qquad \frac{231\ ^{\div 77}}{308\ _{\div 77}} = \frac{3}{4}$$

Try These! Simplify first, then multiply.

a). $\dfrac{8}{3} \times \dfrac{-7}{4}$

b). $\dfrac{9}{16} \times \dfrac{14}{3}$

Answers:

a). $\dfrac{8}{3} \times \dfrac{-7}{4} = \dfrac{\cancel{8}^{2}}{3} \times \dfrac{-7}{\cancel{4}_{1}} = \dfrac{2}{3} \times \dfrac{-7}{1} = \dfrac{-14}{3}$

b). $\dfrac{9}{16} \times \dfrac{14}{3} = \dfrac{\cancel{9}^{3}}{\cancel{16}_{8}} \times \dfrac{\cancel{14}^{7}}{\cancel{3}_{1}} = \dfrac{3}{8} \times \dfrac{7}{1} = \dfrac{21}{8}$

How would you complete this question?

$$0.75 \times \dfrac{-1}{8}$$

You could change 0.75 to a fraction $\dfrac{75}{100} = \dfrac{3}{4}$

OR

You could change $\dfrac{-1}{8}$ to a decimal -0.125

Answer: $0.75 \times (-0.125) = -0.09375$

Or

$$\dfrac{3}{4} \times \dfrac{-1}{8} = \dfrac{-3}{32}$$

Section 3.5: Dividing Rational Numbers

Dividing Integers

a). $(-15) \div (-5) = 3$

b). $\dfrac{(-18)}{9} = -2$

$\div$	$-$	$+$
$-$	$+$	$-$
$+$	$-$	$+$

*** remember the rules with the signs!!!

Dividing Decimals

a). $(-5.1) \div 3$ ✏️⟹

$3 \overline{\smash{)}5.1}$ with 1.7

$\begin{array}{r} 1.7 \\ 3\overline{)5.1} \\ \underline{-3} \\ 2\,1 \\ \underline{-2\,1} \\ 0 \end{array}$

Answer: -1.7

b). $\dfrac{(-7.5)}{-5}$ ✏️⟹

$\begin{array}{r} 1.5 \\ 5\overline{)7.5} \\ \underline{-5} \\ 2\,5 \\ \underline{-2\,5} \\ 0 \end{array}$

Answer: 1.5

c). $(-10.5) \div 0.25$ ✏️⟹ $0.25\overline{)10.5}$ ✏️⟹ $25\overline{)1050.}$

$\begin{array}{r} 42 \\ 25\overline{)1050.} \\ \underline{-100} \\ 5\,0 \\ \underline{-5\,0} \\ 0 \end{array}$

You MUST move the decimal two places 0.25 becomes 25
Therefore the 10.5 must also be adjusted and become 1050.

Answer: -42 Don't forget to go back and look at the sign!

Try These!

1. $(-20.4) \div (-6)$

2). $8.42 \div (-2)$

3. $\dfrac{-138}{6}$

4. $(-0.25) \div (-0.3)$

Answers: 1. 3.4 2. -4.21 3. -23 4. $0.8\overline{3}$

Dividing Fractions

When dividing fractions, keep the first fraction the same and multiply by the reciprocal of the second fraction.

$$\frac{-2}{5} \div \frac{3}{10}$$

keep the same   switch to its reciprocal
                                        $\frac{3}{10}$ becomes $\frac{10}{3}$

change to $\times$

$$\frac{-2}{5} \div \frac{3}{10} = \frac{-2}{5} \times \frac{10}{3} = \frac{-20}{15} = \frac{-4}{3}$$

Still reduce to lowest terms

Examples: Calculate. Reduce answers to simplest form where possible.

1. $\dfrac{3}{4} \div -\dfrac{9}{8} = \dfrac{3}{4} \times \dfrac{-8}{9} = \dfrac{-24}{36} = \dfrac{-24 \div 12}{36 \div 12} = \dfrac{-2}{3}$

2. $1\dfrac{1}{4} \div (-3) = \dfrac{5}{4} \div \dfrac{-3}{1} = \dfrac{5}{4} \times \dfrac{-1}{3} = \dfrac{-5}{12}$

*** remember to change mixed numbers to improper fractions and to write whole numbers as fractions over one.

3. $16 \div \dfrac{-4}{5} = \dfrac{16}{1} \times \dfrac{-5}{4} = \dfrac{\cancel{16}^{\,4}}{1} \times \dfrac{-5}{\cancel{4}_{\,1}} = \dfrac{-20}{1} = -20$

Try These!

4. $\dfrac{-2}{9} \div -\dfrac{4}{7}$ 5. $2\dfrac{1}{2} \div \dfrac{25}{14}$ 6. $\dfrac{8}{11} \div -4$

Answers: . 4. $\dfrac{7}{18}$ 5. $\dfrac{7}{5}$ 6. $\dfrac{-2}{11}$

Multiplication and Division Word Problems

1. A plane seats 480 people. If the plane is $\frac{3}{4}$ full, how many people are on the plane?

   Answer: $480 \times \frac{3}{4} = \frac{480}{1} \times \frac{3}{4} = \frac{1440}{4} = 360$ people on the plane.

2. If a car travels 12.5 km on 1 litre of gas, how many litres of gas does it take to travel 100 km?

   Answer: $100 \div 12.5 = 8\ L$

3. There are 30 people in a row at the movies. How many people are in $5\frac{1}{2}$ rows?

   Answer: $30 \times 5\frac{1}{2} = \frac{30}{1} \times \frac{11}{2} = \frac{330}{2} = 165$ people at the movies.

4. The temperature drops 10.5°C over a 6 hour period. What was the hourly drop in temperature, assuming the temperature dropped the same amount each hour?

   Answer: $10.5 \div 6 = 1.75$ °C drop per hour.

5. A room measures 2.3 m and 3.4 m.   a). What is the area of the room?

   Answer: $2.3 \times 3.4 = 7.82$ m²

b). If carpet cost $18.25/m², calculate the cost to carpet the room. (Exclude Taxes).

   Answer: $7.82 \times 18.25 = \$142.72$

6. Suppose you find $\frac{1}{3}$ of a pizza in the fridge and you eat $\frac{1}{2}$ of it. What fraction of the whole pizza have you eaten?

   Answer: $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ of the whole pizza

7. A tub contains 2.3 L of ice cream. It is shared equally among 5 people. How much will each person get?

   Answer: $2.3 \div 5 = 0.46$ L of ice cream

Section 3.6 Order of Operations with Rational Numbers

B   Do the operations in brackets first

E   Next, evaluate any exponents

D
} Then, divide and multiply in order from left to right
M

A
} Finally, add and subtract in order from left to right
S

Order of Operations with Decimals

Example # 1   **$(-2.4) \div 1.2 - 7 \times 0.2$**

                                      Divide First

       $= -2 - \mathbf{7 \times 0.2}$ Then, multiply

       $= -2 - 1.4$ To subtract, add the opposite

       $= -2 + (-1.4)$

       $= -3.4$

Example # 2   **$(-3.4 + 0.6) + 4^2 \times 0.2$**

                                        Brackets First

       $= -2.8 + \mathbf{4^2} \times 0.2$ Then evaluate the power

       $= -2.8 + \mathbf{16 \times 0.2}$ Then multiply

       $= -2.8 + 3.2$

       $= 0.4$

Order of Operations with Fractions

Example # 1 $\qquad \left(\dfrac{3}{4} - \dfrac{7}{8}\right) \div \left(-\dfrac{5}{16}\right)$

Subtract in the brackets first

Use a common denominator of 8

$$\left(\dfrac{6}{8} - \dfrac{7}{8}\right) \div \left(-\dfrac{5}{16}\right)$$

$$\left(-\dfrac{1}{8}\right) \div \left(-\dfrac{5}{16}\right)$$

To divide, multiply by the reciprocal

$$\left(-\dfrac{1}{8}\right) \times \left(-\dfrac{16}{5}\right)$$

Look for common factors

$$\left(-\dfrac{1}{8_1}\right) \times \left(-\dfrac{16^{\,2}}{5}\right)$$

Both factors are negative, so the product is positive.

$$= \dfrac{2}{5}$$

Example #2 $\qquad \left(-\dfrac{2}{3}\right) \times \dfrac{1}{6} + \dfrac{1}{2}$

Multiply First

$$\left(-\dfrac{2^{\,1}}{3}\right) \times \dfrac{1}{6_3} + \dfrac{1}{2}$$

Look for common factors

$$\left(-\dfrac{1}{9}\right) + \dfrac{1}{2}$$

Add.

Use a common denominator of 18.

$$-\dfrac{2}{18} + \dfrac{9}{18}$$

$$= \dfrac{7}{18}$$

Example # 3 $\left(2\frac{1}{3}\right) + \left(1\frac{1}{4}\right) \times \left(-\frac{2}{3}\right)$ Convert mixed numbers to improper fractions

$$\left(\frac{7}{3}\right) + \left(\frac{5}{4}\right) \times \left(-\frac{2}{3}\right)$$ Multiply first

$$\left(\frac{7}{3}\right) + \left(-\frac{10}{12}\right)$$ Add

Use a common denominator of 12

$$\frac{28}{12} + \left(-\frac{10}{12}\right)$$

$$= \frac{18}{12}$$ Convert improper fractions to mixed numbers

$$= 1\frac{6}{12} = 1\frac{1}{2}$$ Always Reduce

Error Questions

1. A student's solution to a problem, to the nearest hundredth, is shown below. The solution is incorrect. Identify the errors. Provide a correct solution.

$(-8.2)^2 \div (-0.2) - 2.9 \times (-5.7)$

$= 67.24 \div (-0.2) - 2.9 \times (-5.7)$
$= 67.24 \div (-0.2) - 16.53$
$= 67.24 \div (16.73)$
~ 4.02

Answer: $(-8.2)^2 \div (-0.2) - 2.9 \times (-5.7)$

       $67.24 \div (-0.2) - 2.9 \times (-5.7)$

       $-336.2 - 2.9 \times (-5.7)$

       $-336.2 - 16.53$

       -352.73

2. Two students were asked to evaluate:

$$(-8) - 2(24 \div (-8))^2$$

Here are their calculations.

Student 1

$(-8) - 2(24 \div (-8))^2$
$= (-10)(24 \div (-8))^2$
$= (-10)(-3)^2$
$= (-10)(9)$
$= -90$

Student 2

$(-8) - 2(24 \div (-8))^2$
$= (-8) - 2(-3)^2$
$= (-8) - (-6)^2$
$= -8 - 36$
$= -44$

Why did both these students get incorrect answers?   What is the correct answer?

Answer:

Student 1

$(-8) - 2(24 \div (-8))^2$
$= (-10)(24 \div (-8))^2$
$= (-10)(-3)^2$
$= (-10)(9)$
$= -90$

Student 2

$(-8) - 2(24 \div (-8))^2$
$= (-8) - 2(-3)^2$
$= (-8) - (-6)^2$
$= -8 - 36$
$= -44$

Student 1 subtracted first. They didn't follow BEDMAS.

Student 2 multiplied 2 and 3 when they should have done the exponent next.

Correct Answer:
$(-8) - 2(24 \div (-8))^2$

$= (-8) - 2(-3)^2$
$= (-8) - 2(9)$
$= (-8) - 18$
$= -26$

3. The following test question was marked out of 3. What mark would you give this student? Justify your answer.

Calculate:
$$\frac{-7}{8} - \frac{3}{4} \div \frac{1}{5} - \frac{1}{4}$$

Student's Answer:

$$= \frac{-7}{8} - \frac{3}{4} \times \frac{1}{5} - \frac{1}{4}$$

$$= \frac{-7}{8} - \frac{3}{20} - \frac{1}{4}$$

$$= \frac{-7}{40} - \frac{3}{40} - \frac{1}{40}$$

$$= \frac{-11}{40}$$

The student might get 1/3. They knew they had to change the divide to a multiply but forgot to reciprocal the second fraction. They also knew they had to get common denominators but didn't use equivalent fractions and adjust the numerators too.

Correct Answer

$$\frac{-7}{8} - \frac{3}{4} \times \frac{5}{1} - \frac{1}{4}$$

$$= \frac{-7}{8} - \frac{15}{4} - \frac{1}{4}$$

$$= \frac{-7}{8} - \frac{30}{8} - \frac{2}{8}$$

$$= \frac{-39}{8}$$